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EVOLUTIONARY GAMES ON NETWORKS: RETHINKING NETWORK RECIPROCITY

Lecture by FABIO DERCOLE

- Modern *Game Theory* began with John Von Neumann and Oskar Morgenstern in the 40s
- Aim: developing mathematical models of conflict and cooperation between *rational* decision-makers
- The simplest formulation: 2x2 symmetric normal-form games

  2x2: 2 *players* 2 *actions*, say C and D

  2x2 *payoff* matrices Π_1 and Π_2 for players 1 and 2

  symmetric: players are interchangeable, i.e., $\Pi = \Pi_1 = (\Pi_2)^T$

  normal form: simultaneous decisions

$$\Pi_1 = \begin{array}{c|cc} & C & D \\ \hline C & R & S \\ D & T & P \end{array} \qquad \Pi_2 = \cdots$$

- 4 classic examples to study ***cooperation*** among *non-related* individuals

altruistic act with a *cost c* to the actor and a *benefit b* to the recipent

(PD) Prisoner's Dilemma worst case for C (D is the best action) 100% cooperation	$T > R > P > S$ $b > b-c > 0 > -c$ $r > r-1 > 0 > -1$	$T+S<2R$ alternative exploitation does worse than C $r = b/c > 1$ benefit-to-cost-ratio or return $r-1 < 2(r-1)$ ✓
(SD) SnowDrift the actor takes part of the benefit	$T > R > S > P$	$T+S<2R$ alternative exploitation does worse than C
(SH) Stag Hunt	$R > T > P > S$	
(HA) Harmony	$R > T > S > P$	

- One-shot games

  strategy: $x = [p \;\; 1-p]^{\mathrm{T}}$ where p is the probability to play C

  pure strategies: $\mathrm{C} = [1 \;\; 0]^{\mathrm{T}}$ and $\mathrm{D} = [0 \;\; 1]^{\mathrm{T}}$ *mixed* strategies: $p \in (0,1)$

  Nash equilibrium: $\bar{x}$ that is best reply to itself, i.e., $x^{\mathrm{T}} \Pi \bar{x} \le \bar{x}^{\mathrm{T}} \Pi \bar{x}$

(PD) Prisoner's Dilemma	$T > R > P > S$	D is the only Nash
(SD) SnowDrift	$T > R > S > P$	$\bar{p} = (S-P)/(S-P+T-R)$ is the only (mixed) Nash
(SH) Stag Hunt	$R > T > P > S$	C and D are both Nash and there are no mixed Nash
(HA) Harmony	$R > T > S > P$	C is the only Nash

- Repeated games (with the same opponent)

  strategy: decision rule that gives the probability to play C as a function of the history of the interaction

  pure s.: always-C and always-D, *mixed deterministic* s.: e.g. periodic-CD and Tit-for-Tat, *mixed stochastic* s.: …

  Tit-for-Tat (TfT) implements *direct reciprocity* and it won Axelrod's (*J Conflict Resolut* 1980) repeated-PD tournament

  Nash strategy: best reply to itself (but, in general, difficult to show!), e.g. always-D is Nash for the repeated-PD

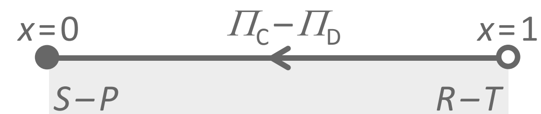
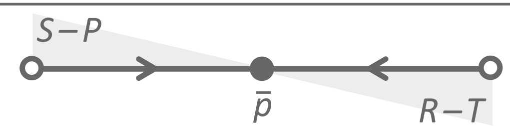
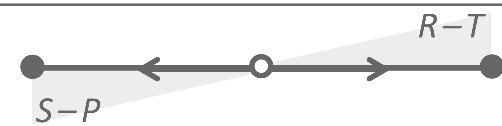
  TfT is also Nash if the probability of a next game is sufficiently high (Axelrod & Hamilton, *Science* 1981)

- *Evolutionary Game Theory* began with John Maynard Smith and George Price in the 70s
- Aim: describe the evolution of the *frequencies* of a given set of strategies within a *population*
- Classic assumption: large and well-mixed populations

  the frequency $x_i = n_i / \Sigma_j n_j$ is the probability to select an *i*-strategist at random ($\Sigma_j x_j = 1$)
- The *replicator equation* (RE): $\mathrm{d}x_i/\mathrm{d}t = x_i(\Pi_i - \langle\Pi\rangle)$, $\langle\Pi\rangle = \Sigma_j x_j \Pi_j$, $\mathrm{d}x/\mathrm{d}t = x(1-x)(\Pi_\mathrm{C} - \Pi_\mathrm{D})$ for 2 strategies

  biological evolution: birth-death processes $\mathrm{d}n_i/\mathrm{d}t = \Pi_i n_i - d\, n_i$ so that $\mathrm{d}x_i/\mathrm{d}t$ = RE

  socio-economic evolution: imitation process $\mathrm{d}x_i/\mathrm{d}t = x_i \Sigma_j x_j (p_{ji} - p_{ij})$ = RE, with $p_{ij} = 0$ if $\Pi_i \geq \Pi_j$, $p_{ij} = \Pi_j - \Pi_i$ otherwise
- The 4 classic games, always-C (freq. x) vs always-D (freq. $(1-x)$): $\Pi_\mathrm{C} = xR + (1-x)S$, $\Pi_\mathrm{D} = xT + (1-x)P$

(PD) Prisoner's Dilemma	$T > R > P > S$	
(SD) SnowDrift	$T > R > S > P$	
(SH) Stag Hunt	$R > T > P > S$	
(HA) Harmony	$R > T > S > P$	

- *Evolution*: invasion → persistence → fixation
- *Evolutionary stability*: strategy A is ESS against B if B cannot invade A
- Stable frequencies correspond to Nash equilibria for the one-shot game (*folk theorem*)

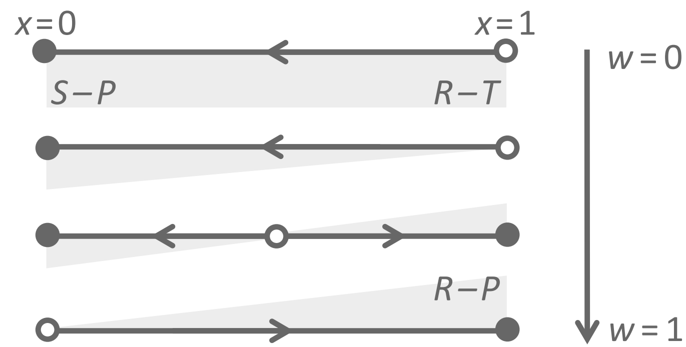
- TfT vs always-D in the PD (w is the re-encounter probability after each encounter)

average number of encounters = $\sum_{j=0}^{\infty} j\, w^j (1-w) \;=\; 1/(1-w)$

$\Pi_{\mathrm{TfT}} = \big(x R/(1-w) + (1-x)(S + wP/(1-w))\big)(1-w)$

$\Pi_{\mathrm{D}} \;= \big(x(T + wP/(1-w)) + (1-x)P/(1-w)\big)(1-w)$

$\Pi_{\mathrm{TfT}} - \Pi_{\mathrm{D}} \;= x\big(R - T(1-w) - wP\big) + (1-x)\big(S(1-w) + wP - P\big)$



- TfT is ESS against always-D but cannot invade (unless $w = 1$)

- Direct reciprocity is unfeasible in large well-mixed populations (insufficient cognitive and memory *resources*)

- Other mechanisms proposed to enhance cooperation (all demanding *resources*)


direct reciprocity


indirect reciprocity


volunteering

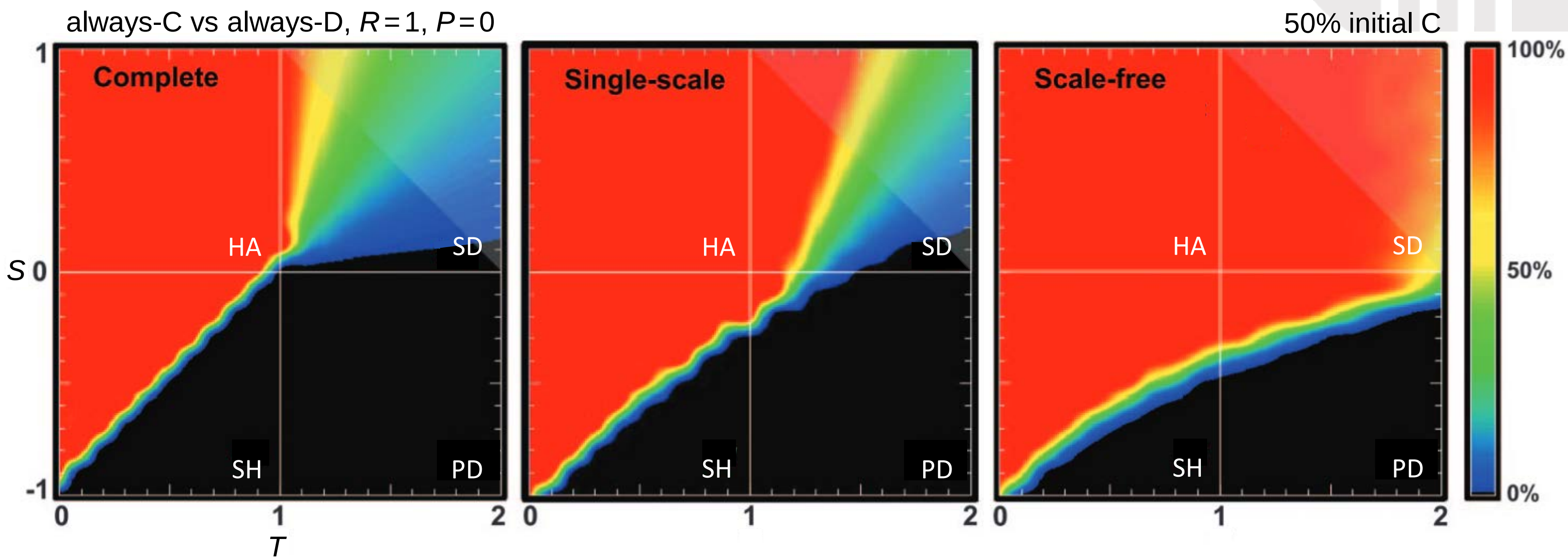

punishment

- *Biological* evolution: birth-death process
- *Socio-economic* evolution: imitation process

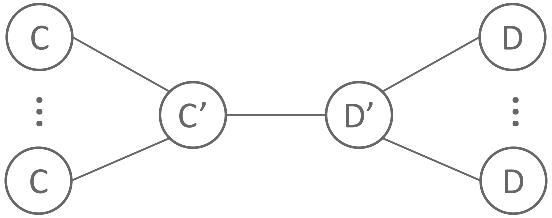
} RE in a large all-to-all network

- Alternatives in non biological context? MPC?

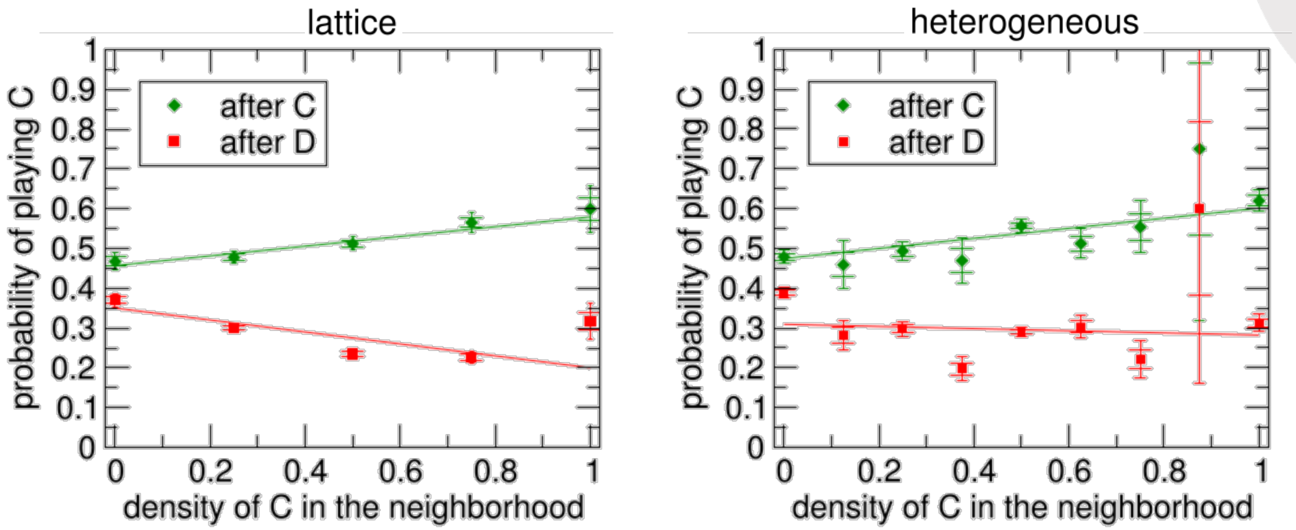
- *Network reciprocity*: a new mechanism enhancing cooperation! Repeated interactions within a local neighborhood support the evolution of C (Nowak & May, *Nature* 1992; Nowak, *Science* 2006)
- *Network heterogeneity* further helps cooperation! (Santos & Pacheco, PRL 2005; Santos et al, *PNAS* 2006)



- Rethinking Network reciprocity:
  - → it works in biological networks and in socio-economic networks under imitation update
  - → it does not explain the invasion of cooperation in the PD
  - → but why should we imitate in socio-economic networks?



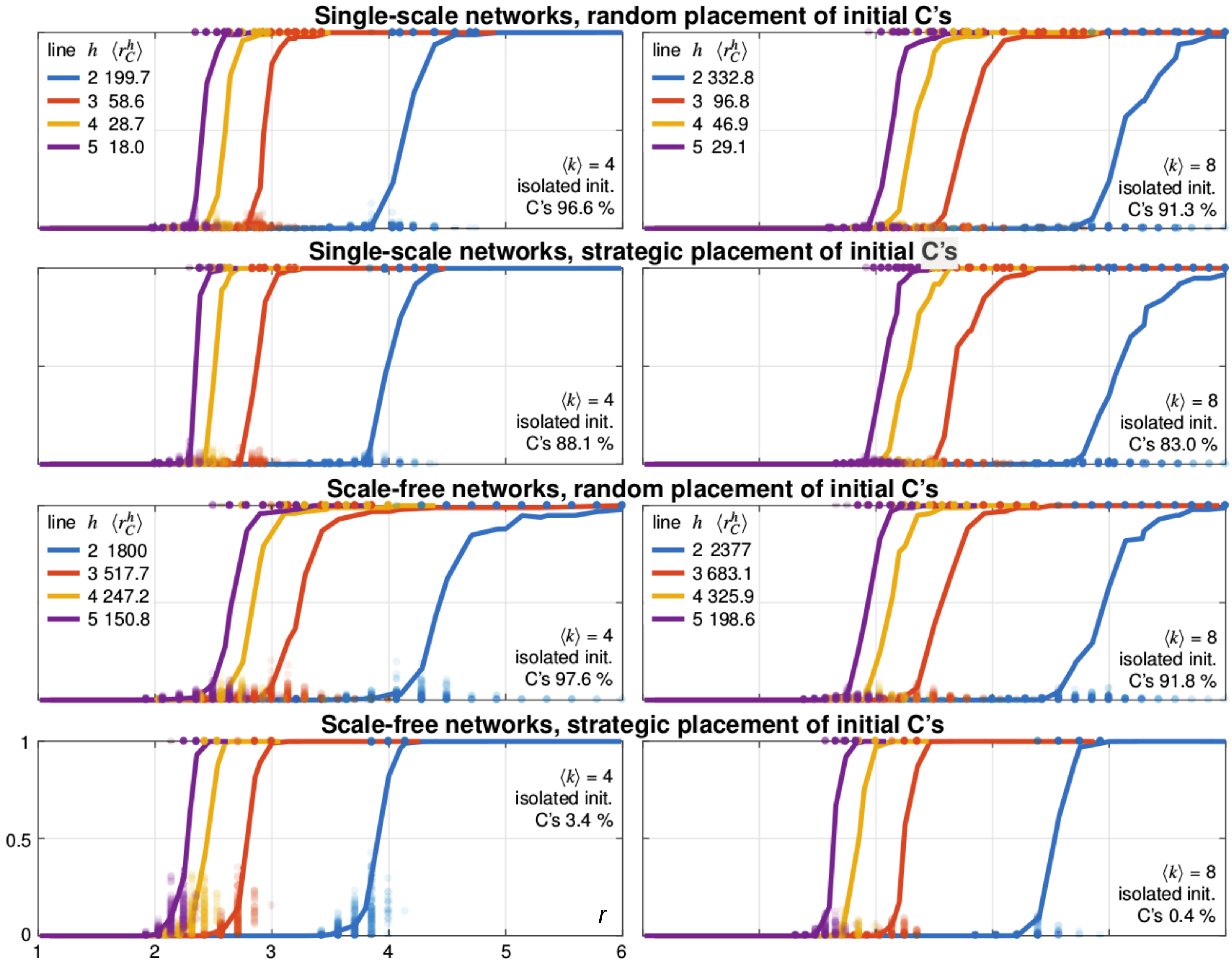
- Repeated PD experiments: Indeed, we do not imitate!
  (Grujić et al, *PLoS ONE* 2010, Gracia-Lázaro et al, *PNAS* 2012)

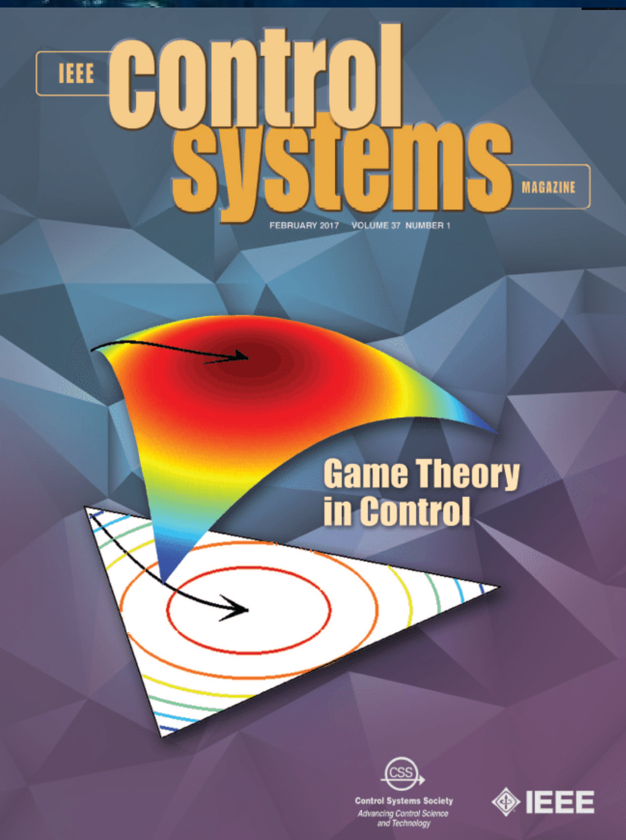


- So, what do we do? Difficult to say… but there seems to be
  → a C or D *mood*
  → a form of *direct reciprocity*

- With no mechanism supporting cooperation, a rational MPC behavior leads to all D in all PD-networks
  → we need to incentivize C and direct reciprocity is the natural way
  → we need a predictive horizon
  → only then, we can study the effect (if any) of the network's structure

- *Networked rational reciprocity* (NRR): local repeated interactions allow direct reciprocity
- A basic MPC-inspired model behavior
  - → at each game round, all individuals play a PD with all neighbors and accumulate payoffs
  - → if exploited by a D-neighbor, a C stops playing with the exploiter for a few rounds
  - → after each round, all individuals independently decide whether to update strategy (with prob δ)
  - → when updating, they change strategy under an expected gain over an horizon of $h \geq 2$ rounds
  - → the expected gain is computed assuming no strategy change within the horizon (δh small)
- Notes
  - → abstention after exploitation is a form of direct reciprocity
  - → how many abstentions? About the time took by the exploiter to update ($1/\delta$ on average)
  - → the number of abstentions is drawn with the prob that the exploiter first updates strategy just after
  - → reciprocity can be modulated by increasing/decreasing the number of abstentions
- Results
  - → for any $h \geq 2$ there is a threshold on r (the PD return) above which C fixates starting from any cluster of C
  - → it works also for an isolated C, provided a D-neighbor first changes to C (prob ~ $1-1/(k+1)$ for small δ)
  - → the threshold is lower in sparse networks
  - → network heterogeneity helps cooperation if the initial C's are strategically placed in the network's hubs (the threshold is higher, but there are good chances that D-leafs change strategy before the hubs)


Single-scale networks, random placement of initial C's
line h $\langle r_C^h \rangle$
2 199.7
3 58.6
4 28.7
5 18.0
$\langle k \rangle$ = 4
isolated init.
C's 96.6 %
line h $\langle r_C^h \rangle$
2 332.8
3 96.8
4 46.9
5 29.1
$\langle k \rangle$ = 8
isolated init.
C's 91.3 %
Single-scale networks, strategic placement of initial C's
$\langle k \rangle$ = 4
isolated init.
C's 88.1 %
$\langle k \rangle$ = 8
isolated init.
C's 83.0 %
Scale-free networks, random placement of initial C's
line h $\langle r_C^h \rangle$
2 1800
3 517.7
4 247.2
5 150.8
$\langle k \rangle$ = 4
isolated init.
C's 97.6 %
line h $\langle r_C^h \rangle$
2 2377
3 683.1
4 325.9
5 198.6
$\langle k \rangle$ = 8
isolated init.
C's 91.8 %
Scale-free networks, strategic placement of initial C's
$\langle k \rangle$ = 4
isolated init.
C's 3.4 %
$\langle k \rangle$ = 8
isolated init.
C's 0.4 %
1
0.5
0
1
2
3
4
5
6
r




LETTER

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Evolutionary dynamics on any population structure

Benjamin Allen[1,2,3], Gabor Lippner[3,4], Yu-Ting Chen[2,3,5], Babak Fotouhi[2,6], Naghmeh Momeni[2,7], Shing-Tung Yau[3,8] & Martin A. Nowak[2,8,9]

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