INTRODUCTION TO NETWORKED DYNAMICAL SYSTEMS

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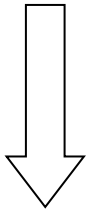
DYNAMICS ON NETWORKS

Each node hosts a (perhaps elementary) dynamical system
[define the local dynamics of each isolated node]

Pairs of dynamical systems interact through the link connecting them
[define the rules of interaction]



What is the collective behavior of the network?
[answer: often more complex (qualitatively) than that of the isolated node]

Does it depend on the topological structure of the network?
[answer: yes, definitely]

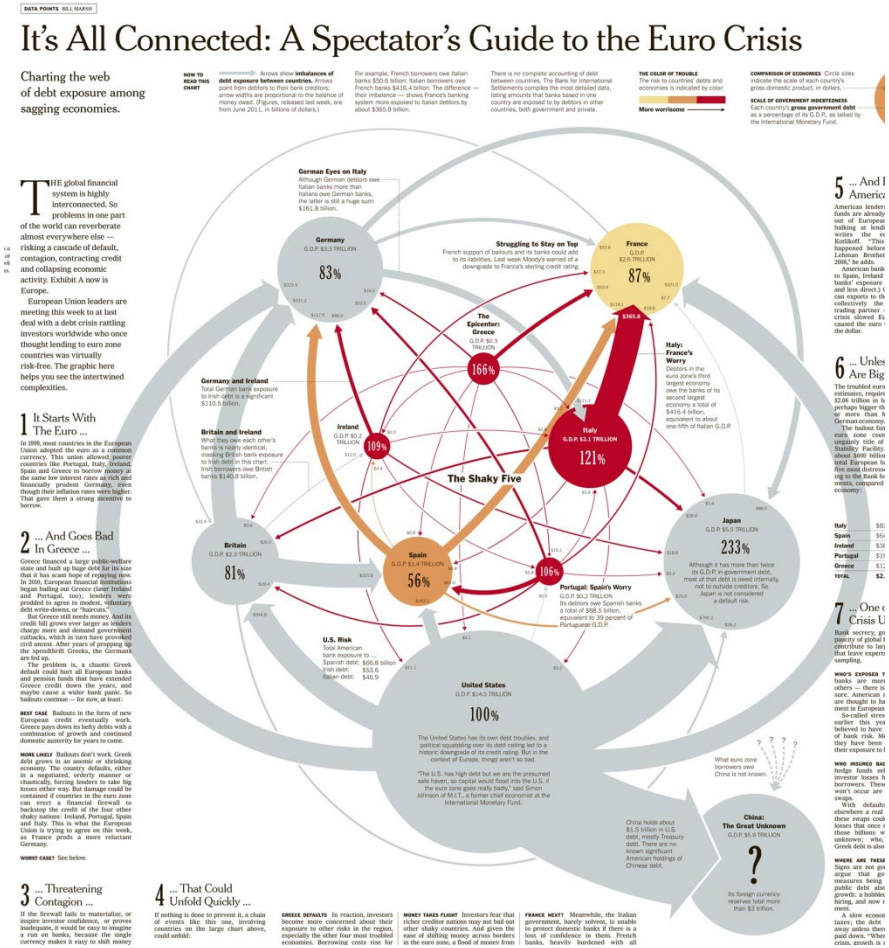
CASCADES OF FAILURES

How breakdown phenomena propagate over the network?

On/off local dynamics (off=overloaded), redistribution of loads if a node goes off.

Applications: power distribution, Internet, financial systems, ...

Figure 1. The 380 kV Italian power transmission network (TERNA 2002, Rosato, Bologna et al. 2007).



CONTAGION AND EPIDEMICS

Probabilistic cellular automata are used to model the spread of infectious diseases over the network - but also of products' adoption, opinions, etc.

- FINITE STATE SET: node (=individual) i is in state $s^i \in \Sigma = \{1,2,\ldots,\sigma\}$ at time t

  e.g.:
  Σ=*{Susceptible, Infected, Recovered}*
  in epidemics
  Σ=*{Non adopter, Adopter}*
  in marketing

- LOCAL RULES (=CONTAGION MECHANISM): the next state s^i_{t+1} depends (according to probabilistic rules) on s^i_t and on the state s^j_t of the neighbors

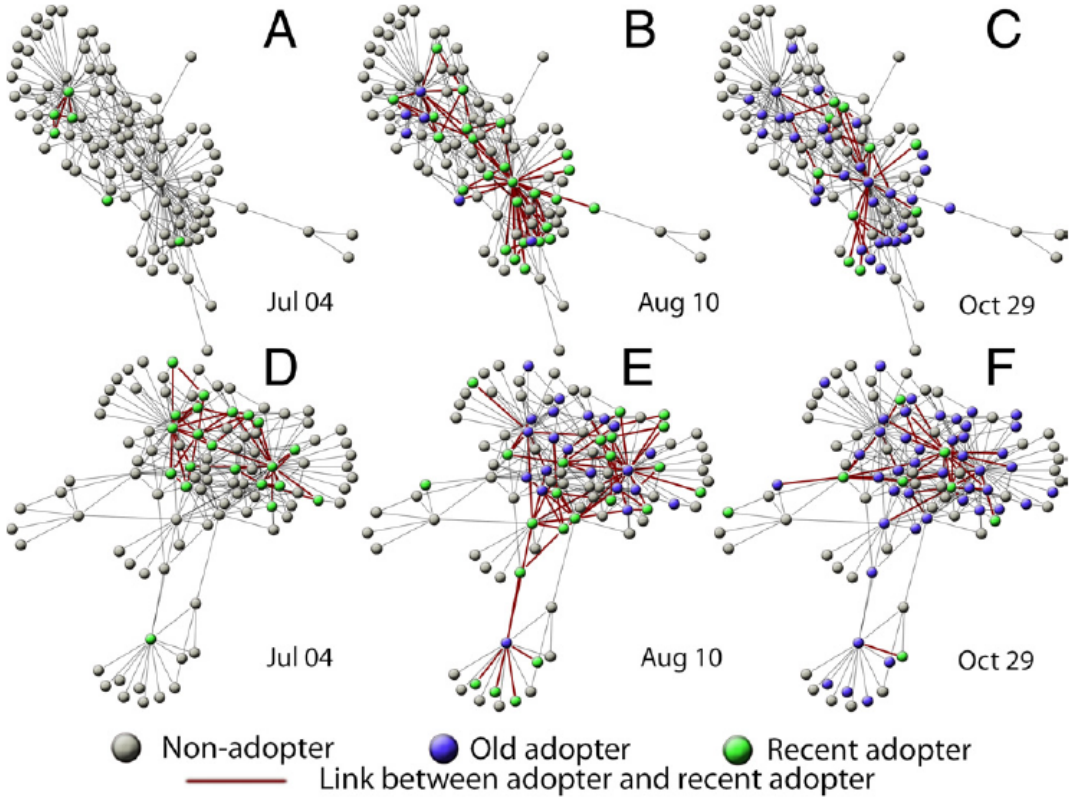


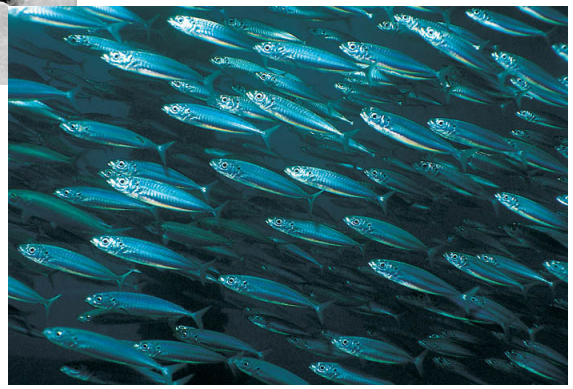
Fig. 1. Diffusion of Yahoo! Go over time. (*A–C* and *D–F*) Two subgraphs of the Yahoo! IM network colored by adoption states on July 4 (the Go launch date), August 10, and October 29, 2007. For animations of the diffusion of Yahoo! Go over time see Movies S1 and S2.

CONSENSUS AND SYNCHRONIZATION

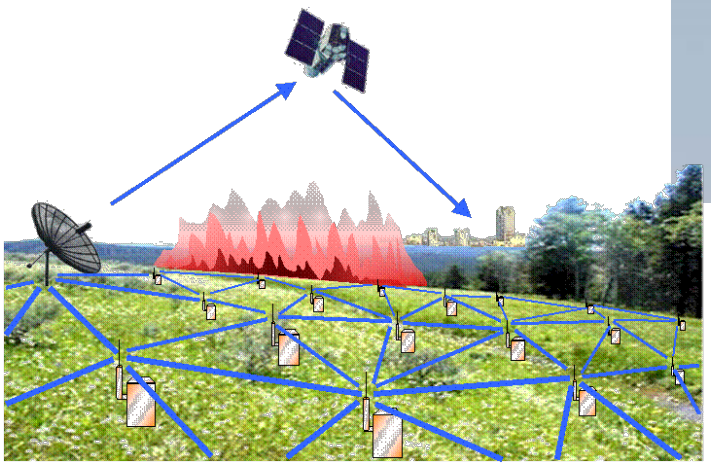
"Distributed consensus"

- a set of agents ($i = 1,2,\ldots,N$)...
- ...reach a common value of a variable ($x_1(t), x_2(t), \ldots \to \bar{x}$)...
- ...by exchanging information only with their neighbors ($i \leftrightarrow j$ iff $a_{ij} = 1$).

Analyzing the consensus phenomena...



...designing the consensus of multi-agent systems



Synchronization = consensus on an oscillatory behavior