

INTRODUCTION TO NETWORKED DYNAMICAL SYSTEMS

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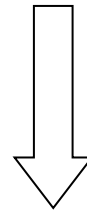
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DYNAMICS ON NETWORKS

Each **node** hosts a (perhaps elementary) **dynamical system**
*[define the **local dynamics** of each isolated node]*

Pairs of dynamical systems **interact through the link** connecting them
*[define the **rules of interaction**]*



What is the **collective behavior** of the network?
*[answer: often **more complex** (qualitatively) than that of the isolated node]*

Does it depend on the **topological structure** of the network?
*[answer: **yes, definitely**]*

CASCADES OF FAILURES

How breakdown phenomena propagate over the network?

On/off local dynamics (off=overloaded), redistribution of loads if a node goes off.

Applications: power distribution, Internet, financial systems, ...



Figure 1. The 380 kV Italian power transmission network (TERNA 2002, Rosato, Bologna et al. 2007).

It's All Connected: A Spectator's Guide to the Euro Crisis

Charting the web of debt exposure among sagging economies.

The global financial system is highly interconnected. So problems in one part of the world can reverberate almost everywhere else — raising a cascade of debt, contagion, contracting credit and collapsing economic activity. Exhibit A now is Europe.

1 It Starts With The Euro ...

In 1999, nine countries in the European Union joined together to form an economic currency. This meant adopting a common currency. This meant adopting a common currency. This meant adopting a common currency.

2 ... And Goes Bad In Greece ...

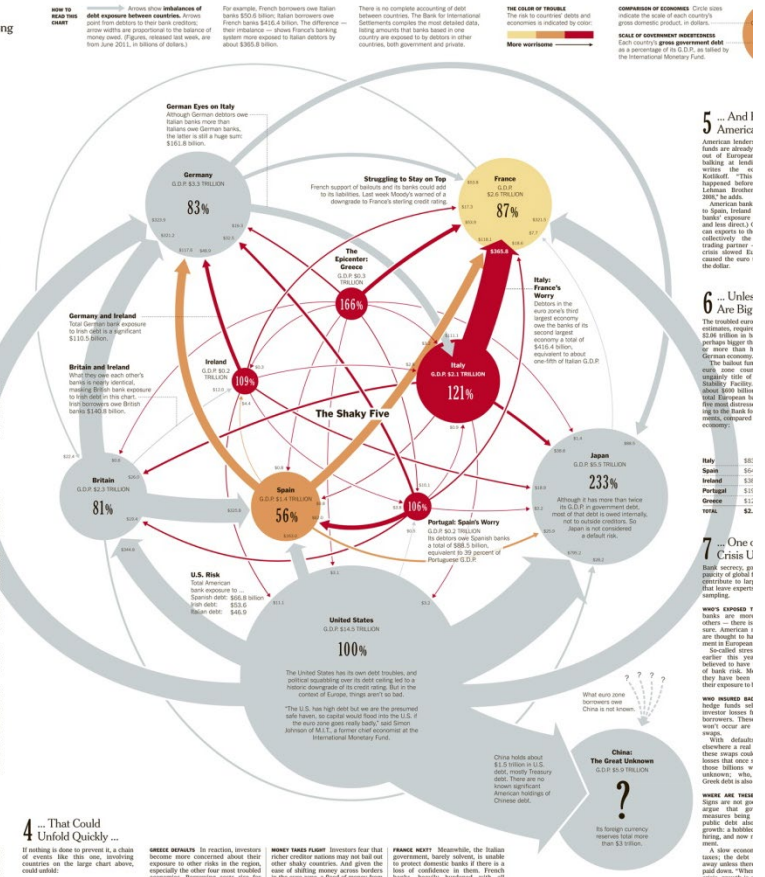
Greece financed a large public welfare state that led to huge debt. It also had a low level of exports. In 2009, European financial institutions began pulling out. Greece's debt rose to 160% of GDP.

3 ... Threatening Contagion ...

If the financial system is interconnected, or people invest in assets, or people trade in assets, or people trade in assets, or people trade in assets.

4 ... That Could Unfold Quickly ...

Investors become more concerned about their exposure to other risks in the region, especially the other four most troubled economies. Borrowers come to the aid of their lenders.



CONTAGION AND EPIDEMICS

Probabilistic cellular automata are used to model the spread of infectious diseases over the network - but also of products' adoption, opinions, etc.

- **FINITE STATE SET:** node (=individual) i is in state $s^i \in \Sigma = \{1, 2, \dots, \sigma\}$ at time t

e.g.:

$\Sigma = \{Susceptible, Infected, Recovered\}$
in epidemics

$\Sigma = \{Non\ adopter, Adopter\}$
in marketing

- **LOCAL RULES (=CONTAGION MECHANISM):** the next state s_{t+1}^i depends (according to probabilistic rules) on s_t^i and on the state s_t^j of the neighbors

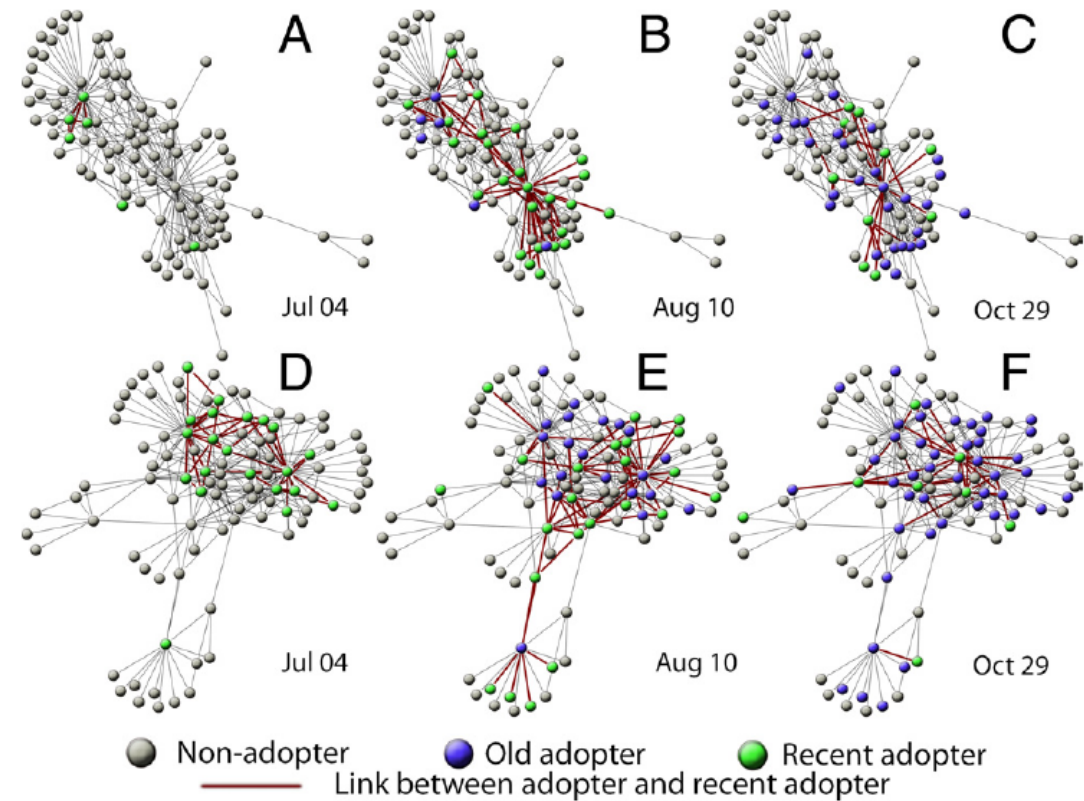


Fig. 1. Diffusion of Yahoo! Go over time. (A–C and D–F) Two subgraphs of the Yahoo! IM network colored by adoption states on July 4 (the Go launch date), August 10, and October 29, 2007. For animations of the diffusion of Yahoo! Go over time see [Movies S1](#) and [S2](#).

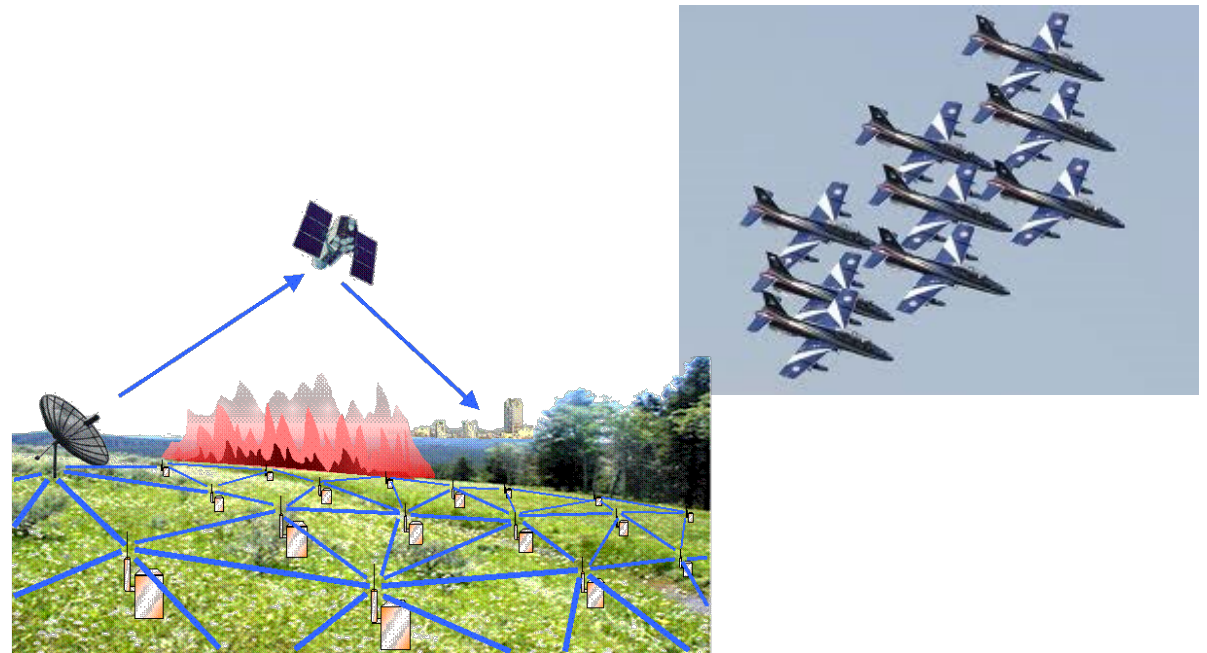
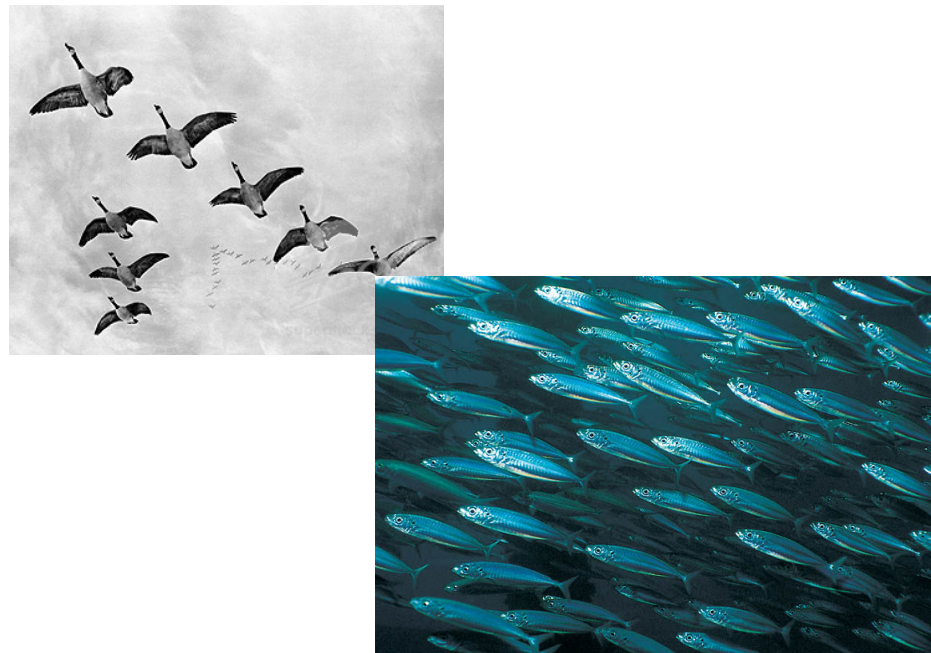
CONSENSUS AND SYNCHRONIZATION

"Distributed consensus"

- a set of **agents** ($i = 1, 2, \dots, N$)...
- ...reach a **common value of a variable** ($x_1(t), x_2(t), \dots \rightarrow \bar{x}$)...
- ...by **exchanging information only with their neighbors** ($i \leftrightarrow j$ iff $a_{ij} = 1$).

Analyzing the consensus phenomena...

...designing the consensus of multi-agent systems



Synchronization = consensus on an **oscillatory behavior**