Advanced Topics in Network Analysis

Link prediction, Recommender Systems, Advanced Network Models

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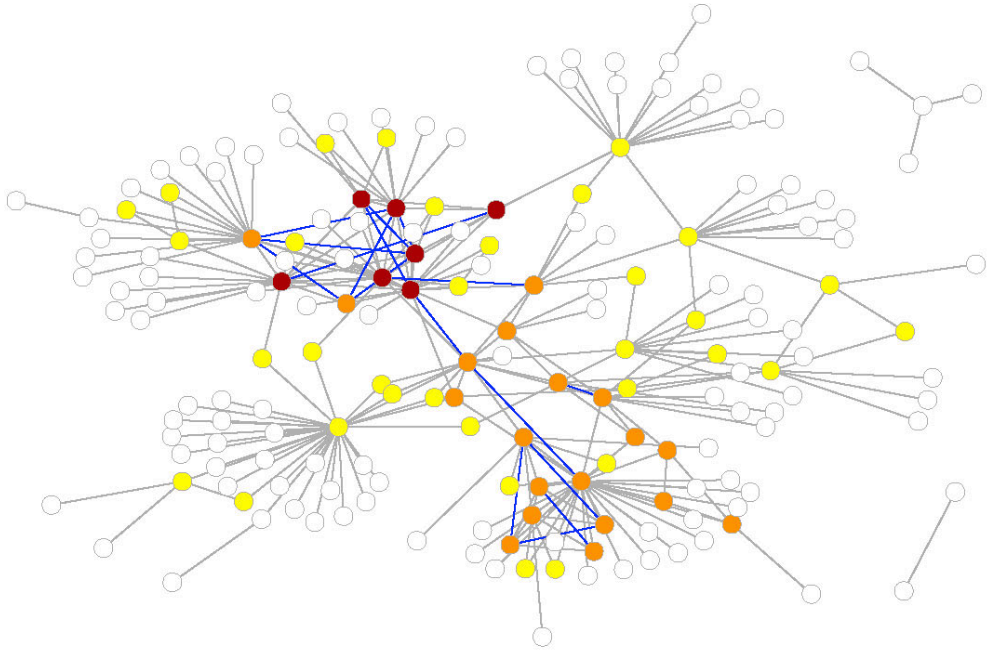
LINK PREDICTION

Networks are used to model interactions observed in field/laboratory experiments, or obtained through data processing, etc.

Many existing interactions might be overlooked (also, a few observed interactions might be artificial).

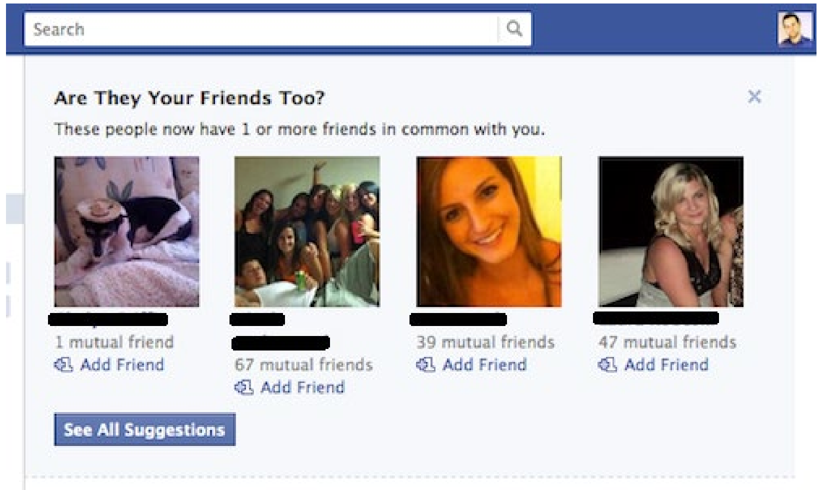
Main assumption: the observed network A^O is a "noisy" observation of a true network A^{true}.

Problem: reconstruct A^{true} given A^O.



Applications:

- find missing links (perhaps suggesting ad-hoc experiments)
- delete spurious links
- in time-varying networks, predict future links (e.g., who will be your future friends?)



Finding missing links: Similarity-based algorithms

The observed network A^O (undirected, unweighed) has N nodes $(i = 1,2,\dots,N)$ and L observed links $(a_{ij} = 1)$.

S is the set of non observed links $(a_{ij} = 0)$, whose cardinality is $\left(\frac{N(N-1)}{2} - L\right)$.

Assumption: the set S contains missing links (=existing but not observed).

The more two nodes are "similar", the more is the likelihood they are connected

- define a similarity score s_{ij} for each node pair (i,j)
- define a threshold $\bar{s}$
- a non observed link is a missing link if $s_{ij} > \bar{s}$
- *(optional: an observed link is a spurious link if $s_{ij} < \bar{\bar{s}} < \bar{s}$)*

Similarity

Node attributes

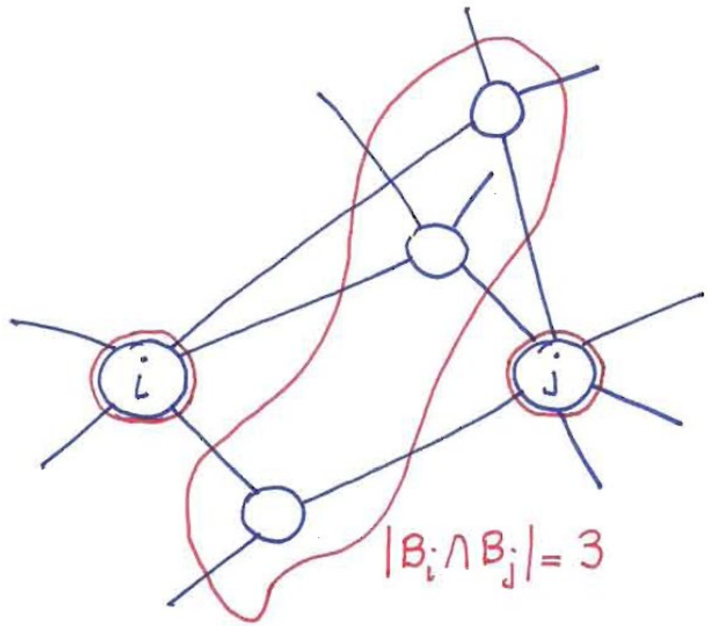
$\boxed{\textit{Network topology}}$

How to quantify the structural (topological) similarity of nodes (i, j) ?

- Number of common neighbours: $s_{ij} = |B_i \cap B_j| = (A^2)_{ij}$=*n. of length-2 paths*
- ... many variations (normalizations e.g. by functions of the degrees k_i,k_j)
- generalization: consider paths (i, j) longer than 2 (Katz index)

$$s_{ij} = \beta A_{ij} + \beta^2 (A^2)_{ij} + \beta^3 (A^3)_{ij} + \cdots = [(I - \beta A)^{-1} - I] \quad , \quad 0 < \beta < 1/\lambda_{max}(A)$$

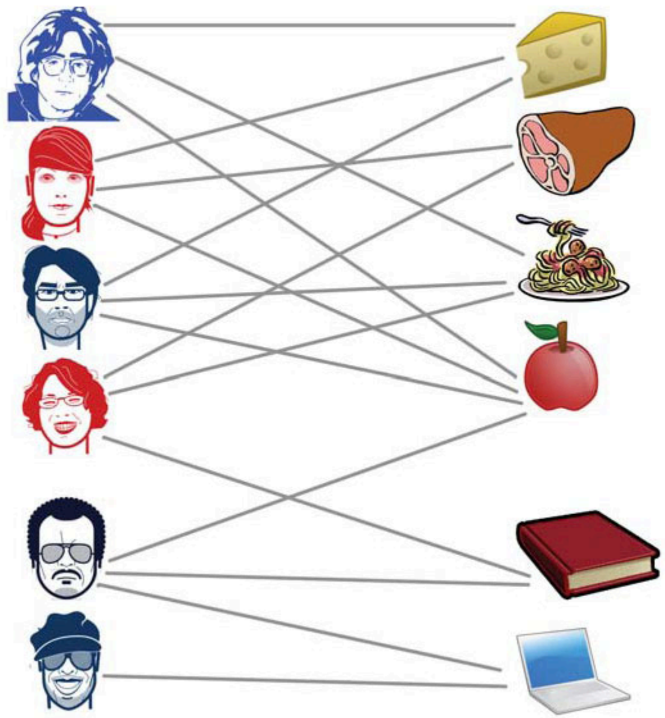
- ... Katz index truncated at some power
- Preferential attachment index: $s_{ij} = k_i k_j$
- Resource allocation index: $s_{ij} = \sum_{h \in (B_i \cap B_j)} {}^1/_{k_h}$
- ...many others



RECOMMENDER SYSTEMS

$U = \{u_1, u_2, \dots, u_m\}$: set of users $\qquad\qquad$ $O = \{o_1, o_2, \dots, o_n\}$: set of objects

The set of objects owned by each user u_i is coded by the bipartite network $B = [b_{ij}]$:



$$b_{ij} = 1 \text{ if } u_i \text{ owns } o_j, \qquad b_{ij} = 0 \text{ otherwise}$$

- user degree (number of objects owned by i):

$$k(u_i) = \sum_j b_{ij}$$

- object degree (number of users owning j):

$$k(o_j) = \sum_i b_{ij}$$

Problem: assuming u_i likes the objects she/he owns (=not purchased at random), predict (and recommend) which more object(s) she/he could like.

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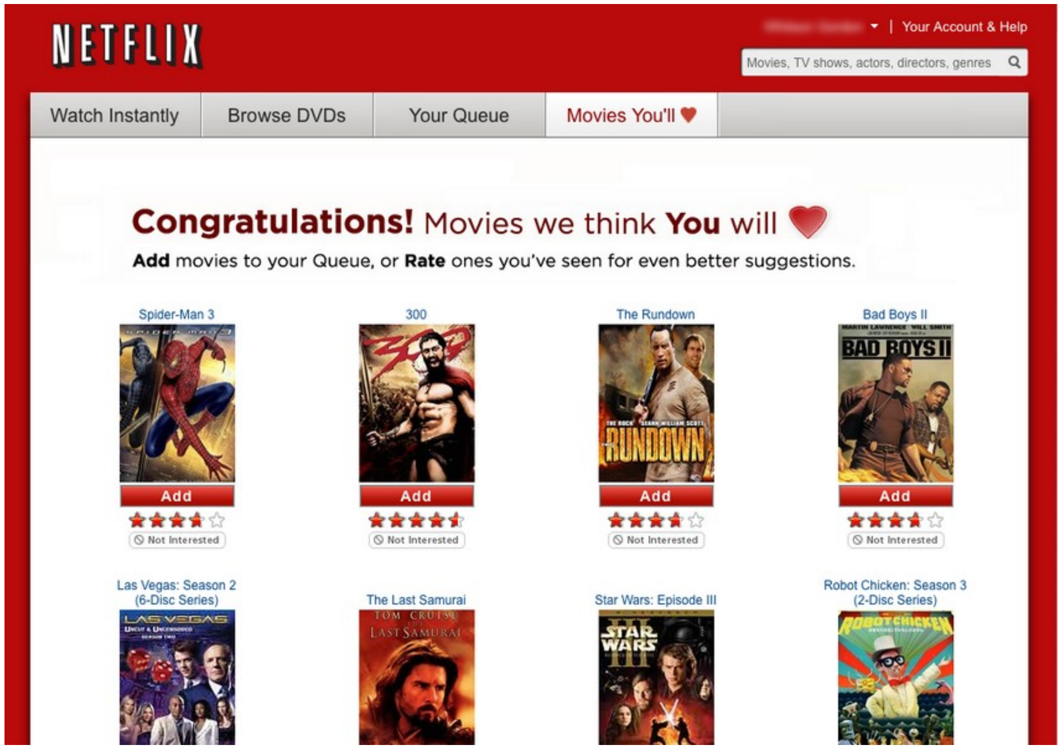

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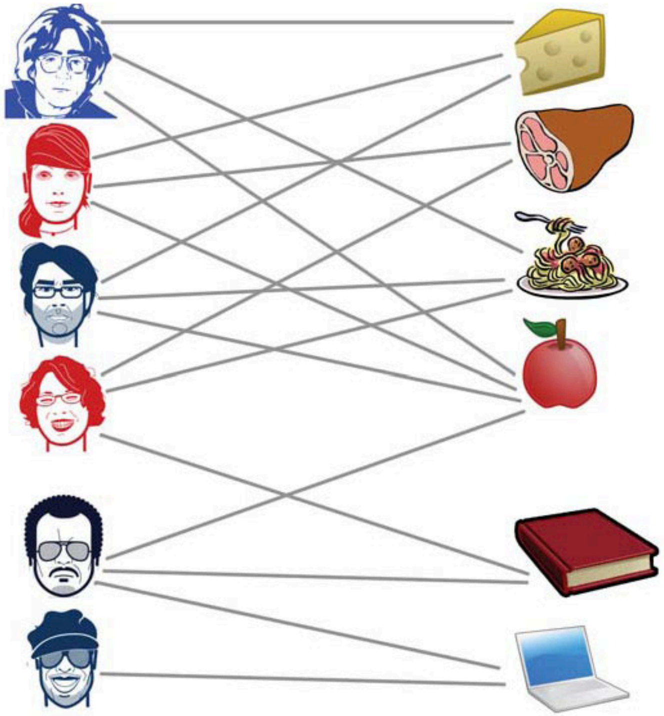
A naive solution (global ranking method): recommend the objects o_j with the largest degree $k(o_j)$ (=the object owned by the largest number of users).



The recommendation list is the same for all users (no personalization):

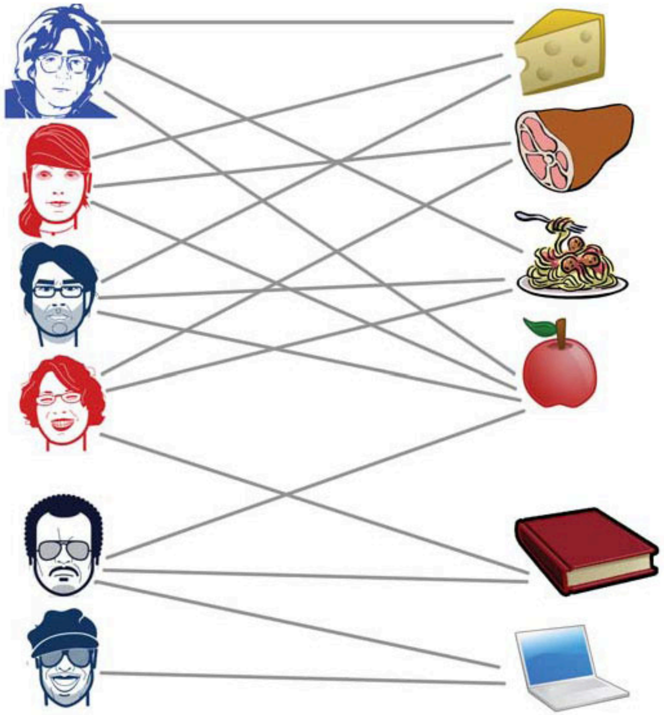
1st: apple ($k = 4$)
2nd: cheese, spaghetti ($k = 3$)
4th: ham, book, laptop ($k = 2$)

The network is not used.

A less naive solution (content based filtering): define a similarity among objects, and recommend to user u_i the objects o_j most similar to those already owned.

This is a personal recommendation system – still the network is not used.

In collaborative filtering, the main assumption is that similar users like similar objects.



The score assigned to the (non-existing) pair (u_i, o_j):

$$\hat{b}_{ij} = \frac{s_{i1}b_{1j} + s_{i2}b_{2j} + \cdots + s_{im}b_{mj}}{s_{i1} + s_{i2} + \cdots + s_{im}} = \frac{\sum_{l=1}^{m} s_{il}b_{lj}}{\sum_{l=1}^{m} s_{il}}$$

This is a link prediction procedure on a bipartite network.

How to quantify the similarity between users i, l ?

Two examples (among the simplest ones) of user similarity:

- topological similarity: common neighbours = n. of objects in common:

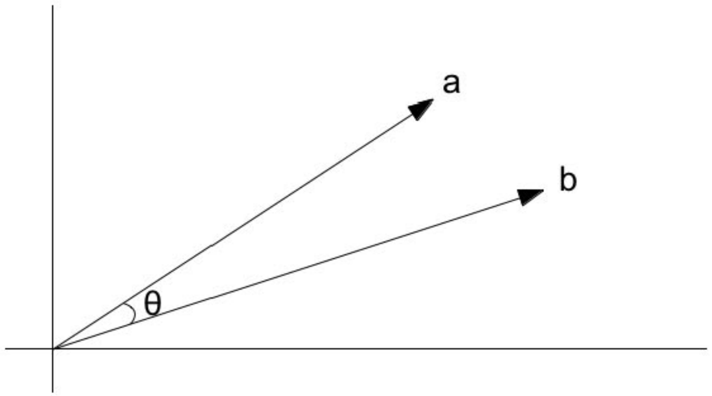
$$s_{il} = \sum_{j=1}^{n} b_{ij} b_{lj}$$

- rating-base similarity: users give a score (e.g., 1 to 5 stars) to the objects they own:

$$r_i = (r_{i1} \quad r_{i2} \quad \dots \quad r_{in})$$

r_{ij} is the score given to object j by user i.

The more the vectors r_i, r_l are close each other (=same preferences), the more users i, l are similar.

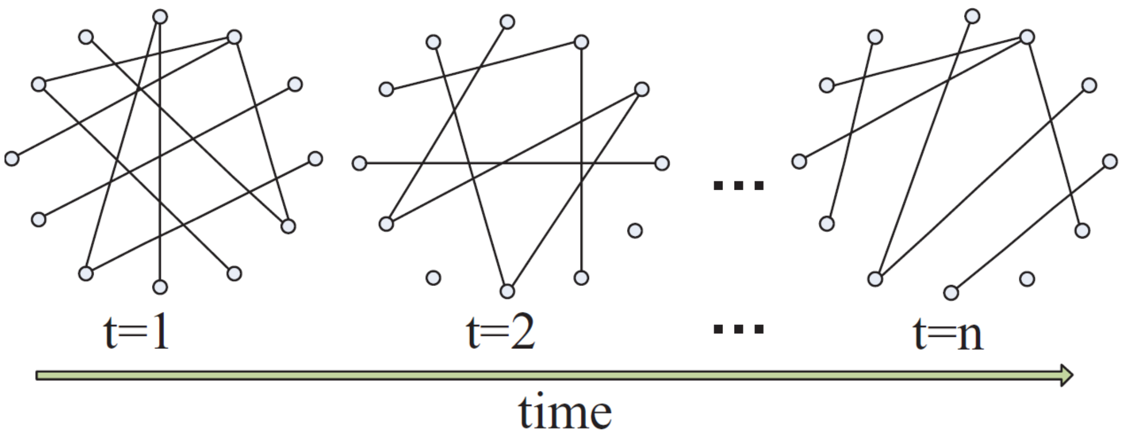


"Cosine" similarity:

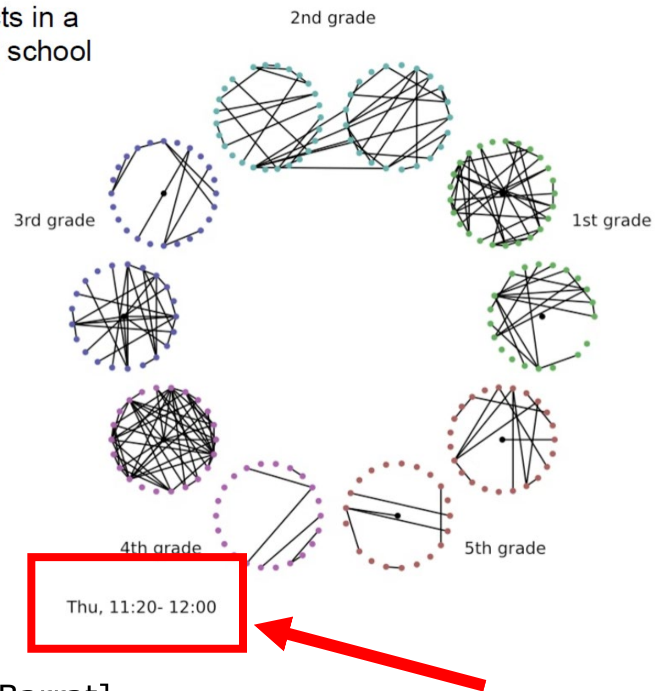
$$s_{il} = \frac{r_i \cdot r_l}{|r_i||r_l|}$$

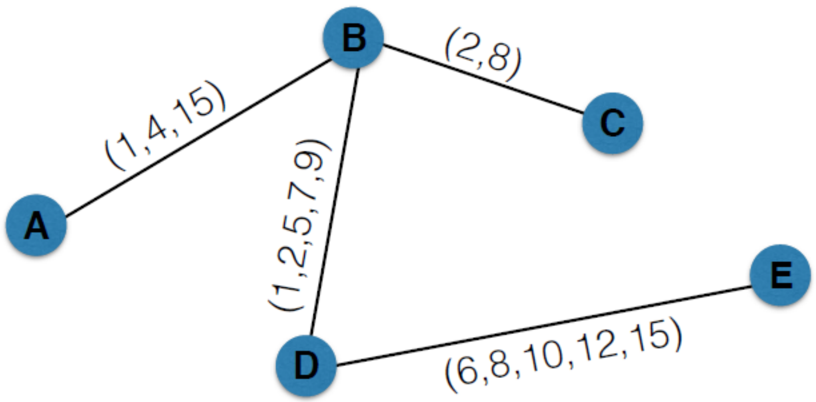
ADVANCED NETWORK MODELS
(JUST A GLANCE...)

TEMPORAL NETWORKS (links vary in time)



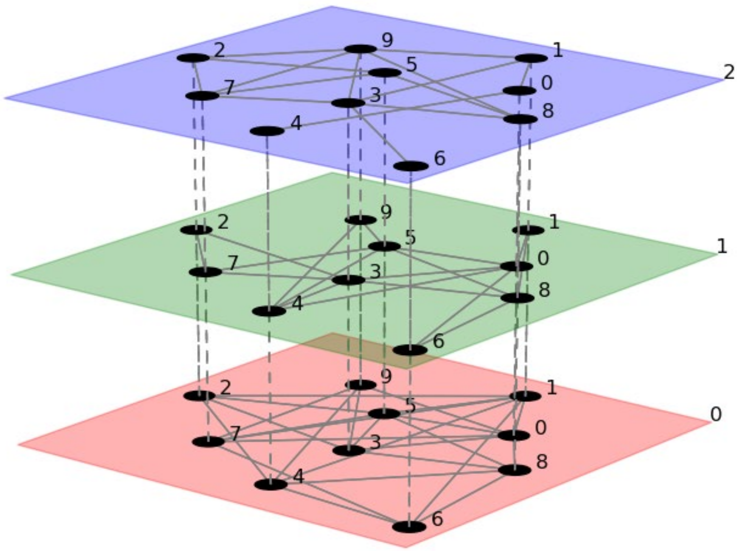
contacts in a primary school

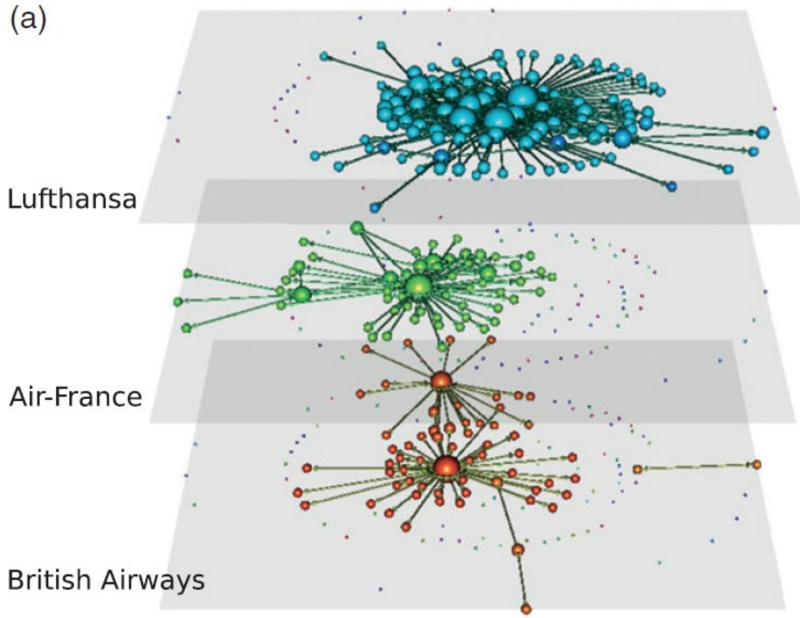




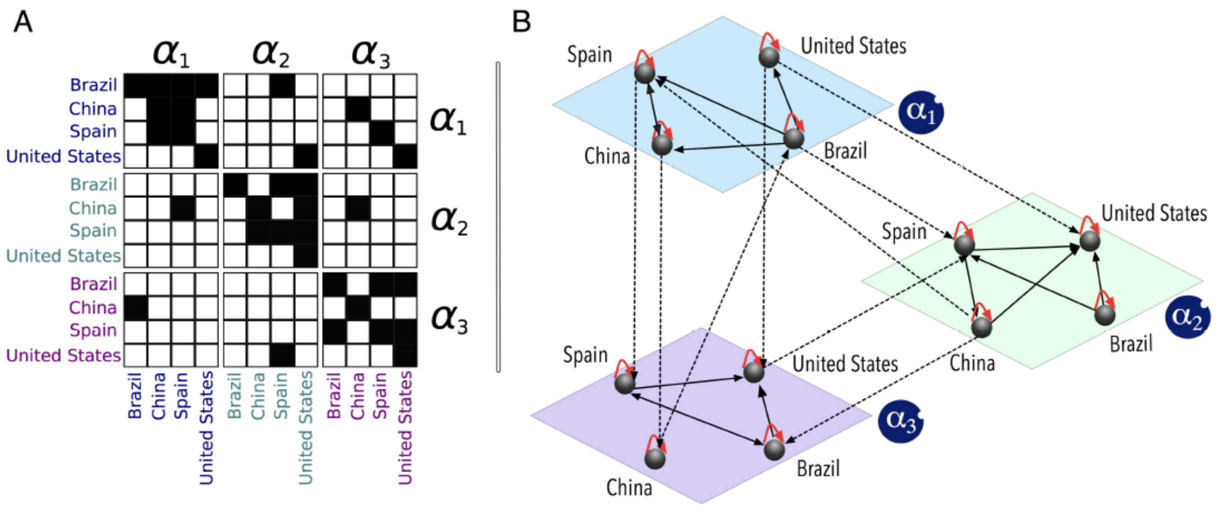
[from Barrat]

[MULTILAYER NETWORKS](#) (nodes are connected by different sets of intra-layer links, plus possibly inter-layer links)



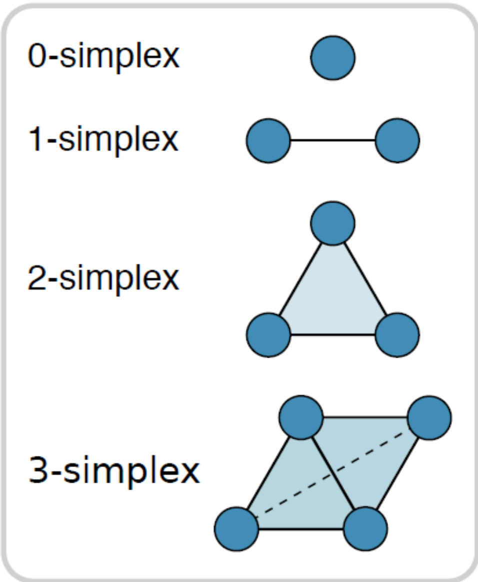


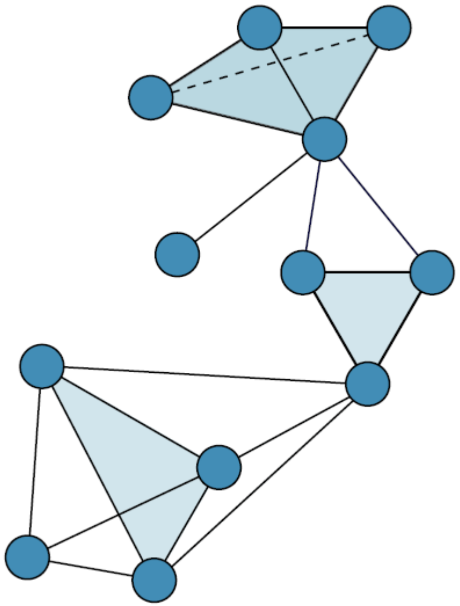
[from Kivela]



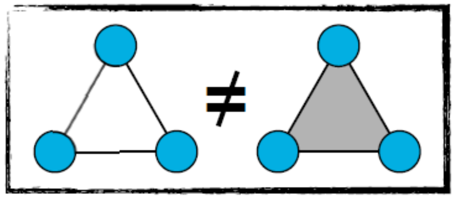
[Alves et al,SciRep 2019]

HIGHER-ORDER INTERACTIONS (HYPERGRAPHS) (node-to-node connections are not only pairwise)

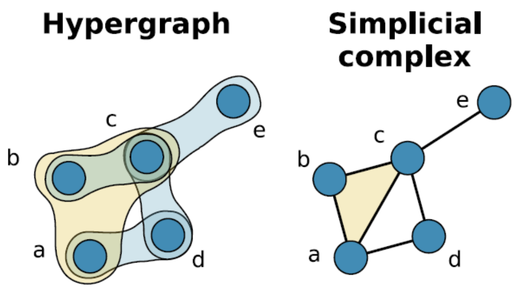


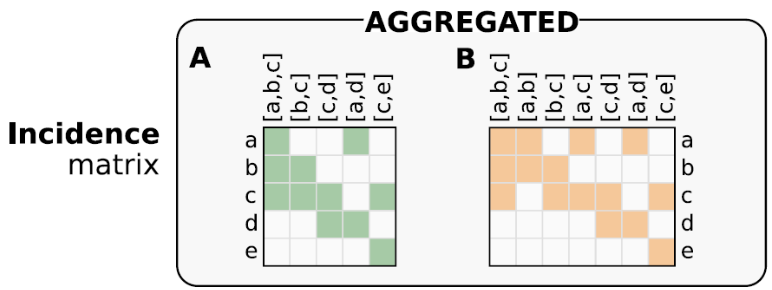


Notice that



[from Petri]





[Battiston et al PhysRep 2020]