NETWORKS AND THEIR REPRESENTATIONS

Carlo PICCARDI

DEIB - Department of Electronics, Information and Bioengineering
Politecnico di Milano, Italy

email carlo.piccardi@polimi.it
https://piccardi.faculty.polimi.it/

Not logged in Talk Contributions Create account Log

Article Talk Read Edit View history Search

WIKIPEDIA
The Free Encyclopedia

Main page
Contents
Featured content
Current events
Random article
Donate to Wikipedia
Wikipedia store

Interaction
Help
About Wikipedia
Community portal
Recent changes
Contact page

Tools

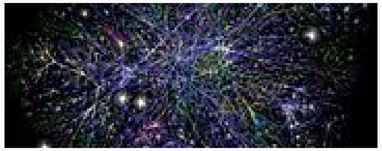
Complex network

From Wikipedia, the free encyclopedia
(Redirected from Complex networks)

In the context of network theory, a **complex network** is a graph (network) with non-trivial topological features—features that do not occur in simple networks such as lattices or random graphs but often occur in graphs modelling of real systems. The study of complex networks is a young and active area of scientific research (since 2000) inspired largely by the empirical study of real-world networks such as computer networks, technological networks, brain networks and social networks.

Contents [hide]

Network science



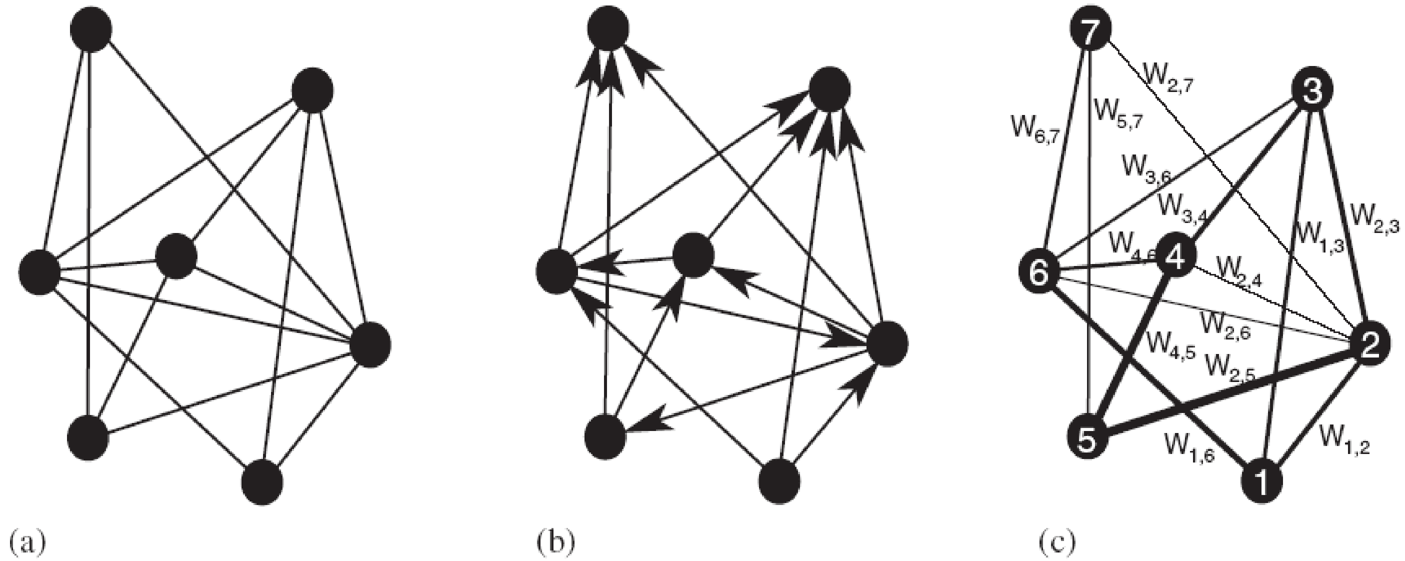
Theory

Graph · **Complex network** · Contagion · Small-world · Scale-free · Community structure · Percolation · Evolution · Controllability · Graph drawing · Social capital · Link analysis · Optimization · Reciprocity · Closure · Homophily · Transitivity · Preferential attachment · Balance theory · Network effect · Social influence

NETWORKS

A network is represented by a graph with N nodes (or vertices) and L links (or edges).

Nodes represent individuals, objects, subsystems, etc..
Links represent interactions, dependencies, communication channels, etc.



A network can be undirected (a,c) or directed (b); weighted (c) or unweighted (binary) (a,b).

Networks provide a truly interdisciplinary modeling tool...

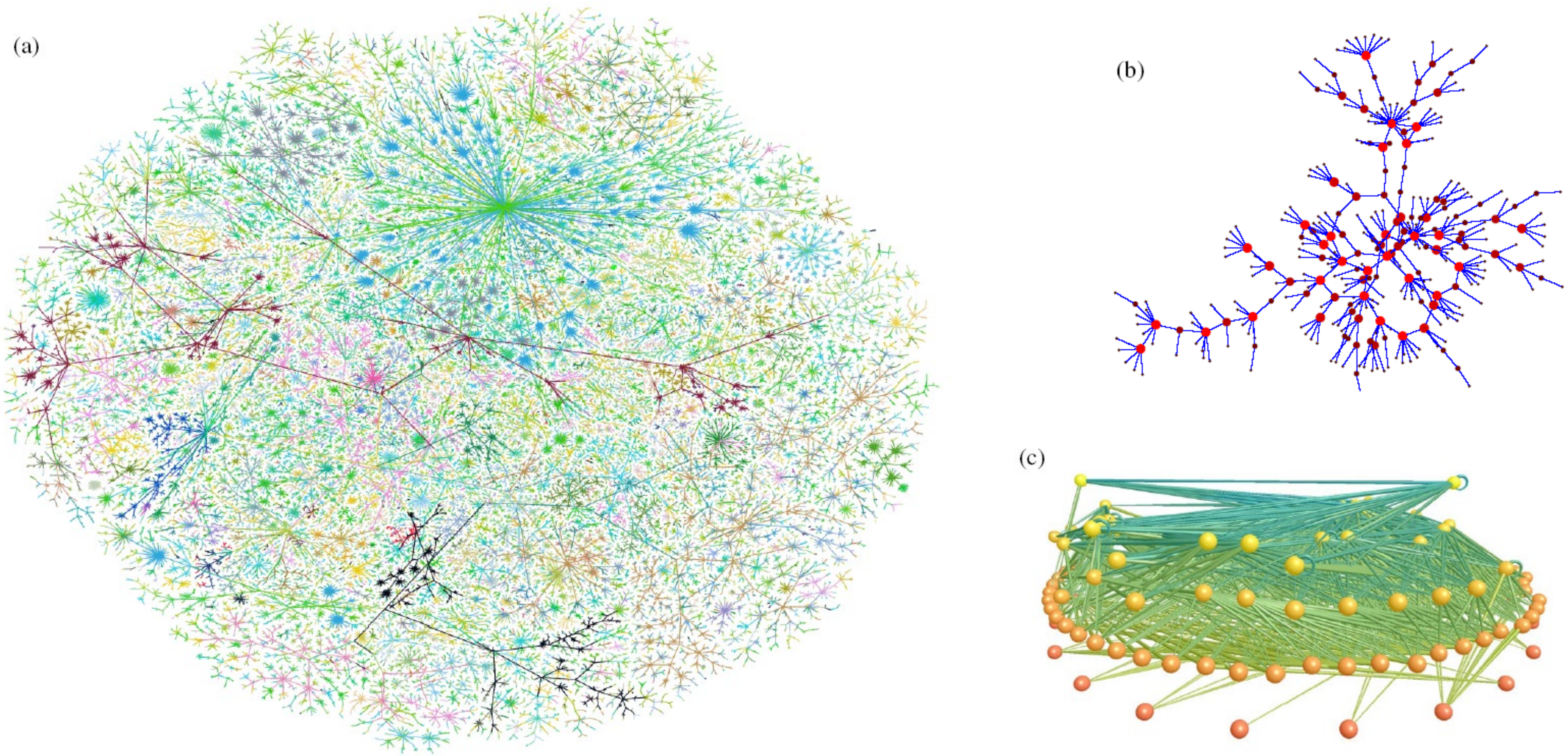


Fig. 1.2 *Three examples of the kinds of networks that are the topic of this review.* (a) *A visualization of the network structure of the Internet at the level of "autonomous systems"—local groups of computers each representing hundreds or thousands of machines. Picture by Hal Burch and Bill Cheswick, courtesy of Lumeta Corporation.* (b) *A social network, in this case of sexual contacts, redrawn from the HIV data of Potterat et al.* [341]. (c) *A food web of predator-prey interactions between species in a freshwater lake* [271]. *Picture courtesy of Richard Williams.*

Social networks...

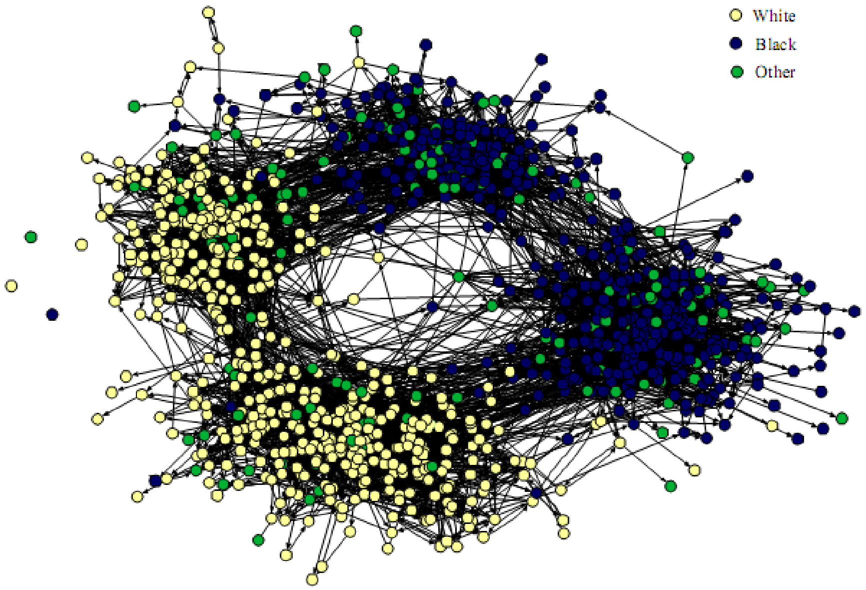
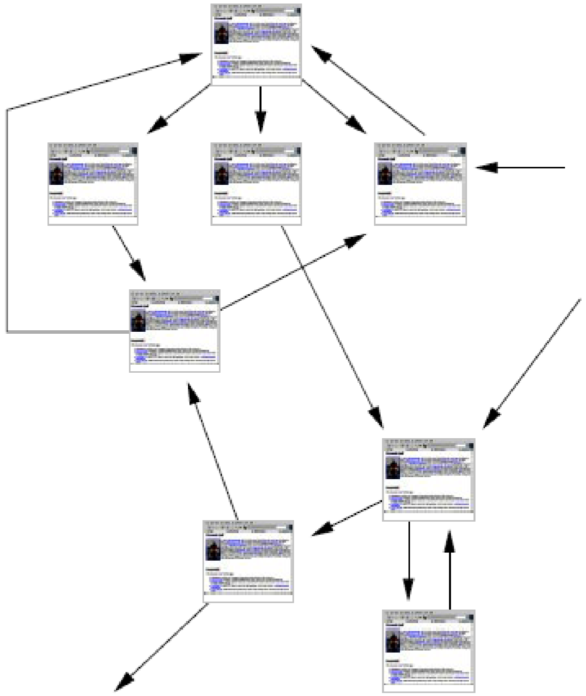
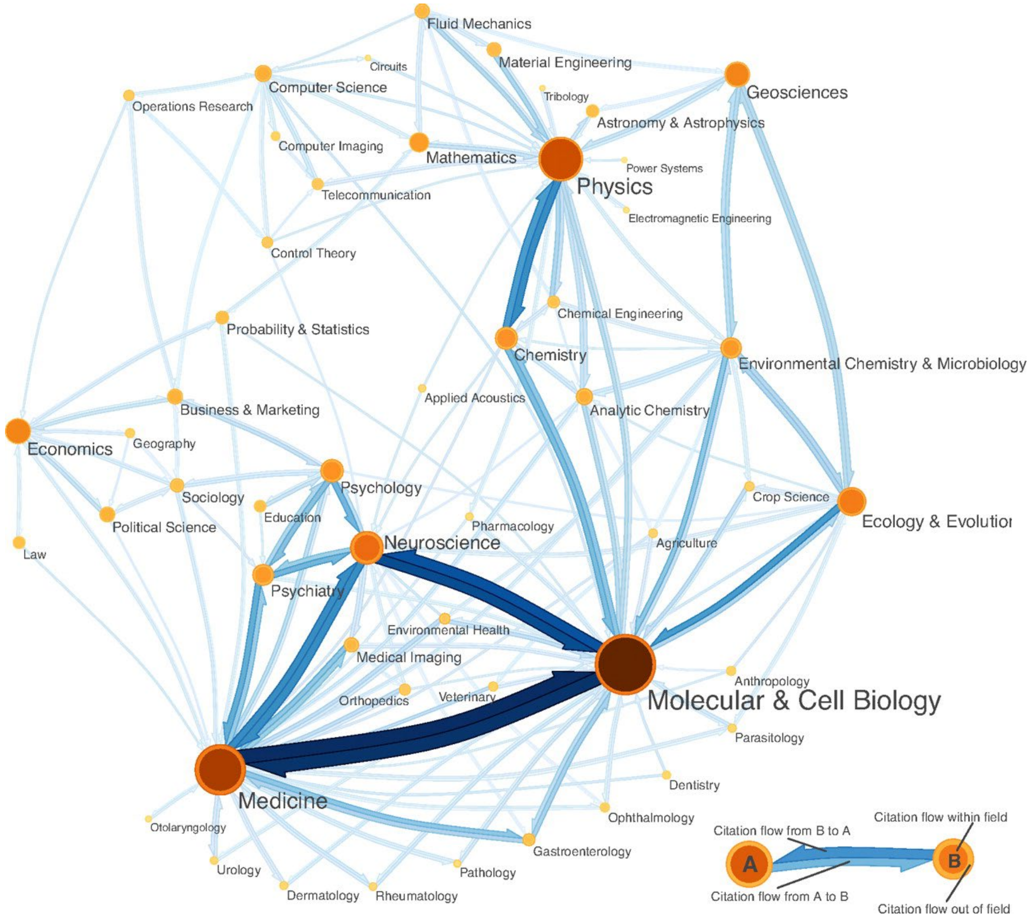


Fig. 3.4 *Friendship network of children in a U.S. school. Friendships are determined by asking the participants, and hence are directed, since A may say that B is their friend but not vice versa. Vertices are color coded according to race, as marked, and the split from left to right in the figure is clearly primarily along lines of race. The split from top to bottom is between middle school and high school, i.e., between younger and older children. Picture courtesy of James Moody.*

Information networks...



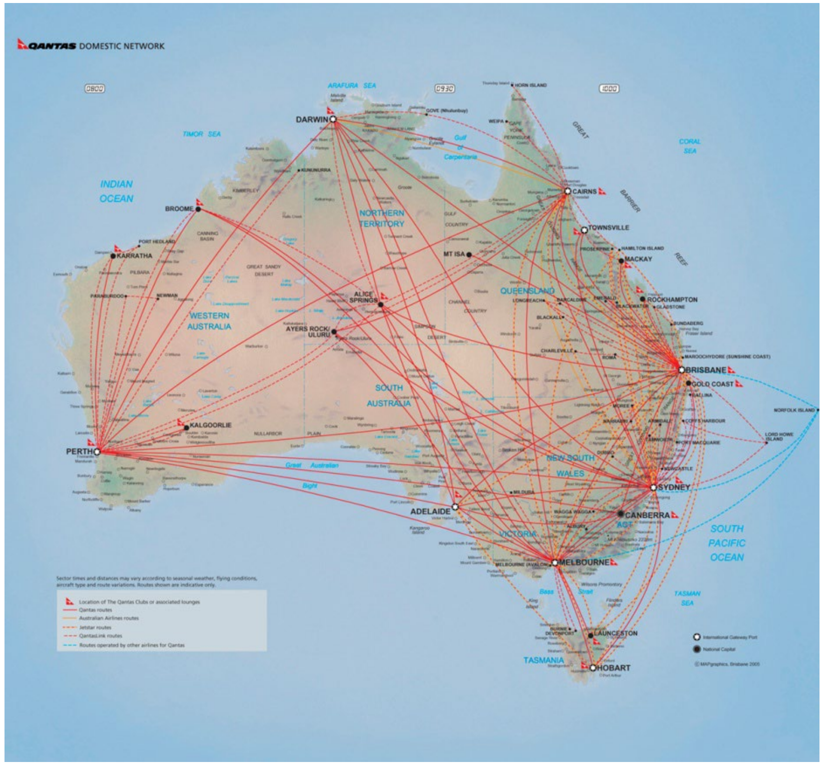
World–Wide Web

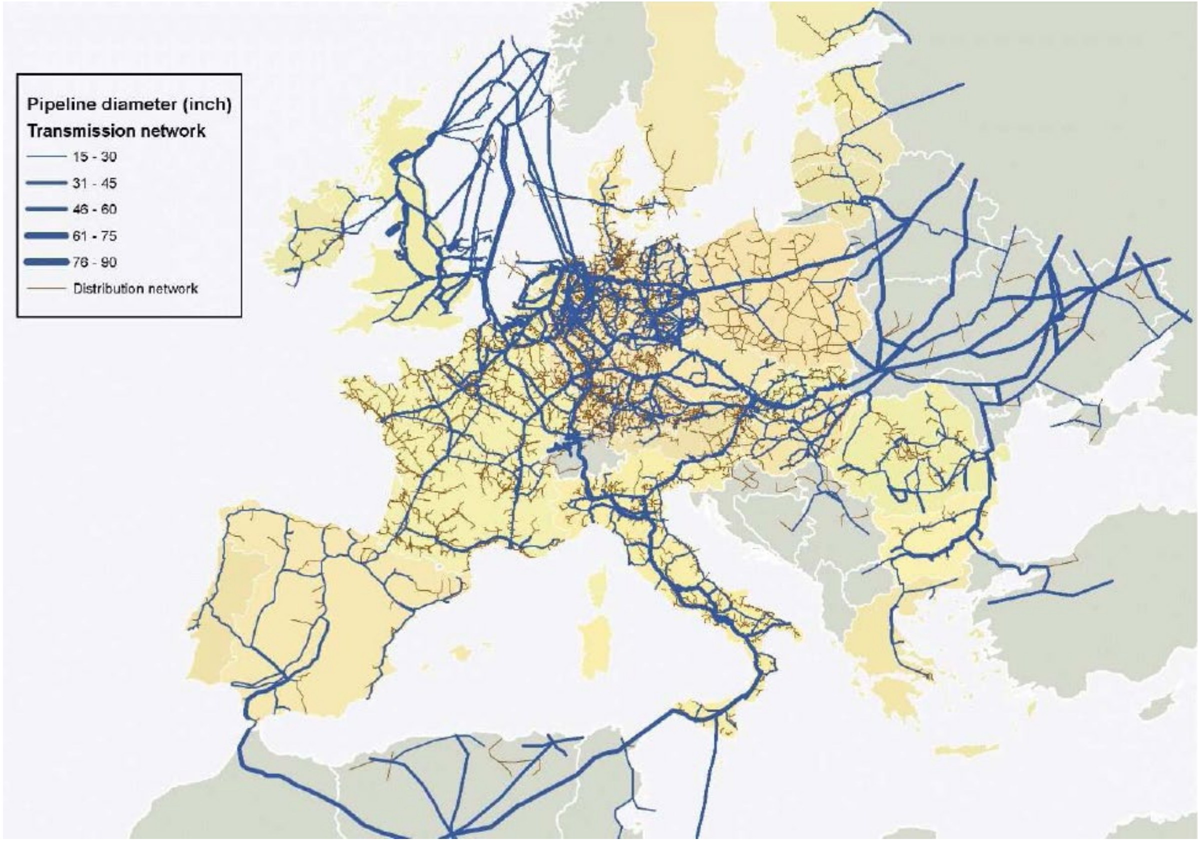


Rosvall and Bergstrom 2008

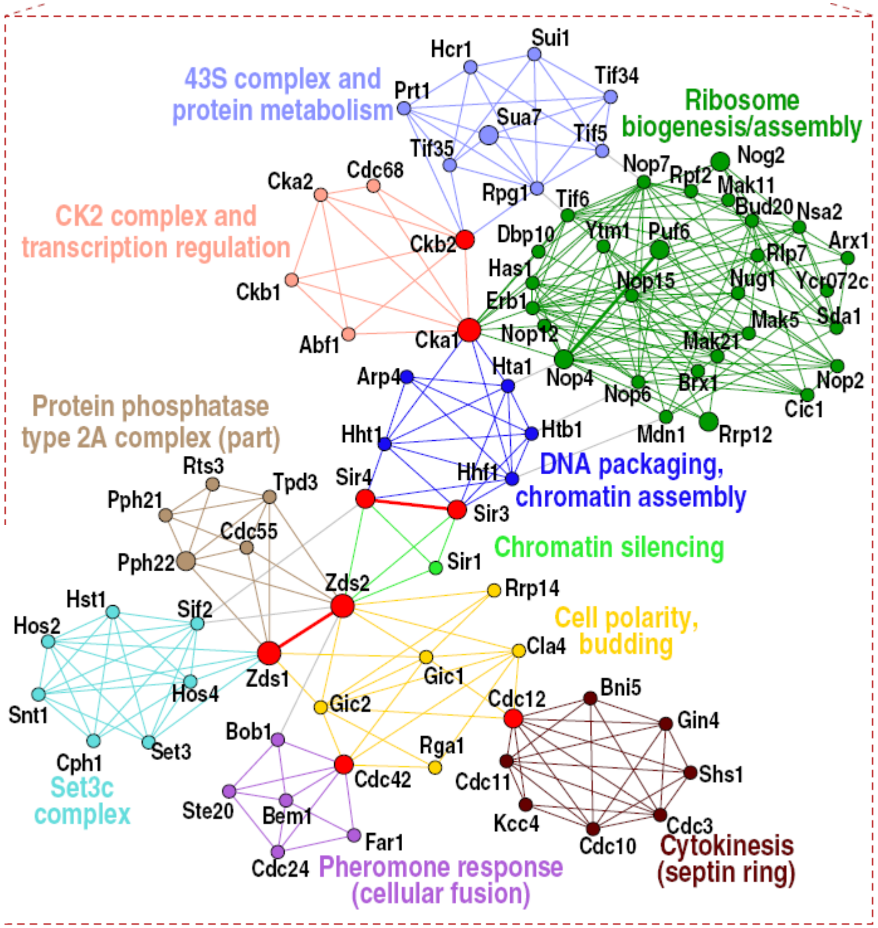
journal-to-journal citation network

Transportation networks...

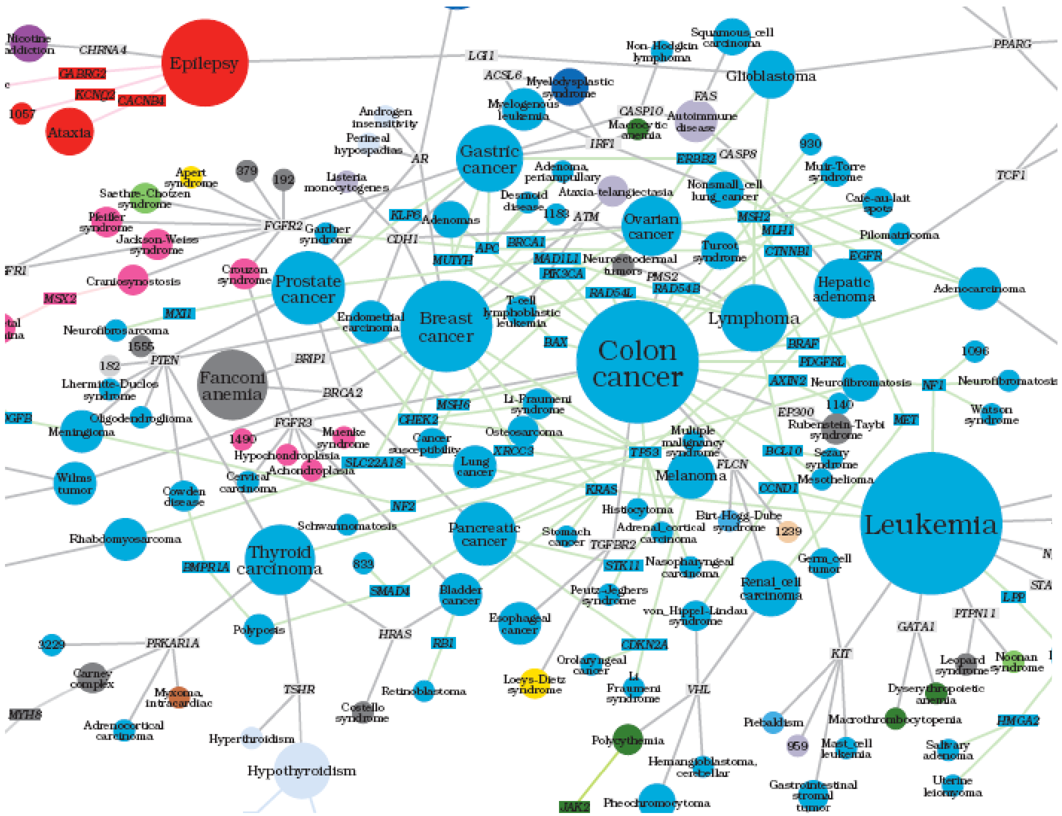




Biological networks...



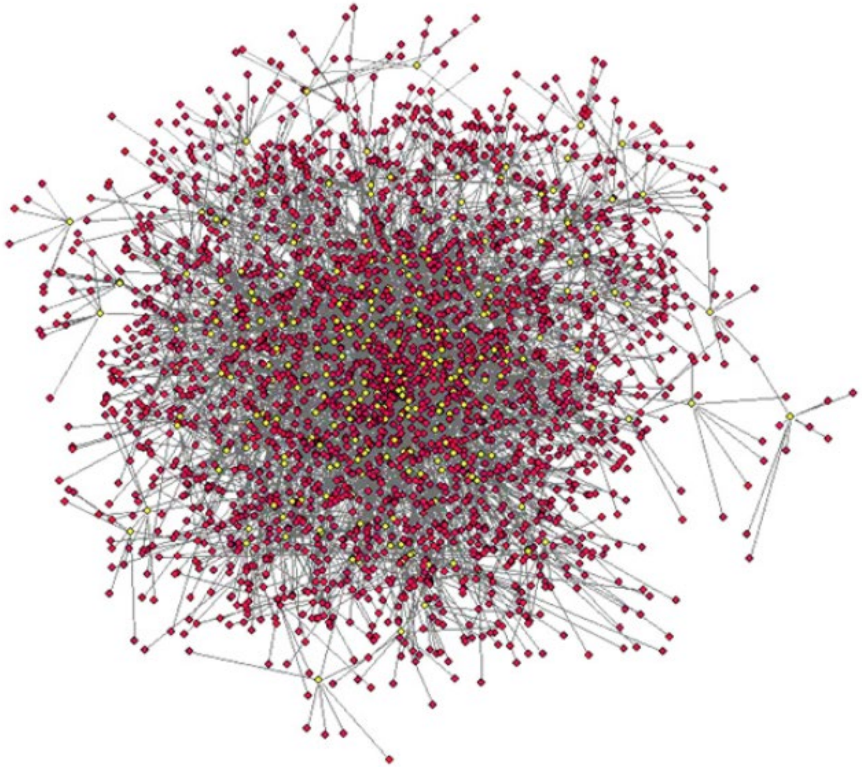
Palla et al, Nature, 2005



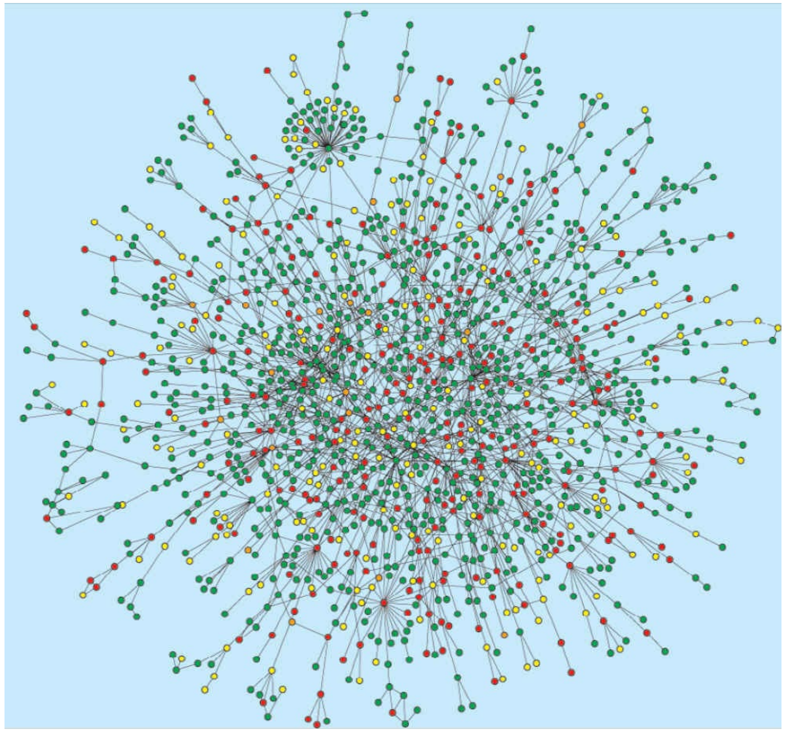
Goh et al, PNAS, 2007

Similar problems are found in very different contexts:

⇨ common theories, methods, algorithms



The "directors network" of the Italian companies

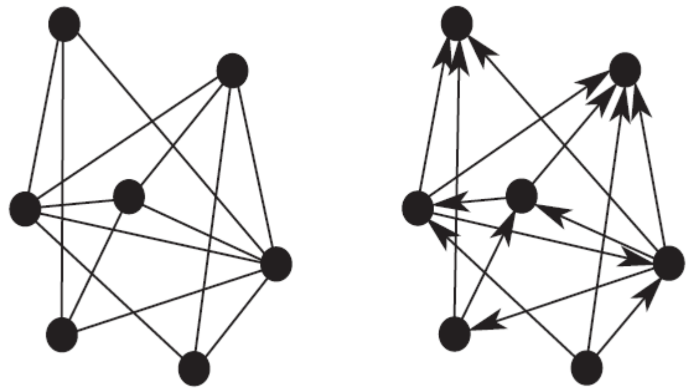


The protein interaction network of yeast

<u>ADJACENCY MATRIX</u>

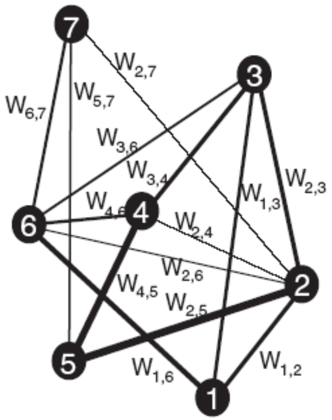
An unweighted network is completely described by the $N \times N$ adjacency matrix $A = [a_{ij}]$:

$$a_{ij} = 1 \text{ if the link } i \rightarrow j \text{ exists,}$$
$$a_{ij} = 0 \text{ otherwise}$$



A is symmetrical if the network is undirected, asymmetrical if the network is directed.

Typically, A is a sparse matrix: small density $\rho = \frac{L}{N(N-1)}$ (dir.) or $\rho = \frac{L}{N(N-1)/2}$ (undir.).

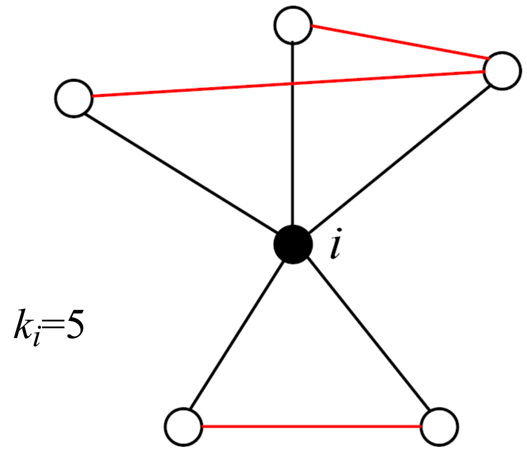


A weighted network is described by the $N \times N$ weight matrix $W = [w_{ij}]$:

$$w_{ij} > 0 \text{ if the link } i \rightarrow j \text{ exists,} \quad w_{ij} = 0 \text{ otherwise}$$

In an undirected network, the degree k_i of node i is the number of links connected to i (=the number of neighbors of i):

$$k_i = \sum_j a_{ij}$$



The $N \times N$ Laplacian matrix

$$L = diag(k_1, k_2, \ldots, k_N) - A$$

is an alternative network representation, where:

$$l_{ii} = k_i \quad , \quad i = 1,2,\ldots,N$$

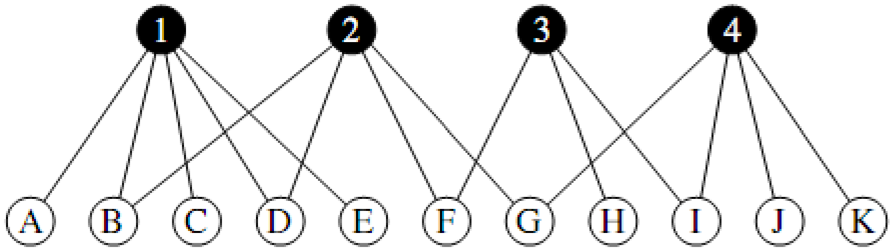
$l_{ij} = -1$ if the link $i \leftrightarrow j$ exists, $l_{ij} = 0$ otherwise $(i \neq j)$

L is symmetric and zero-row-sum, and $(-L)$ is a Metzler matrix $\Rightarrow$ spectral properties

BIPARTITE ("TWO-MODE") NETWORKS

They are composed of two distinct classes of nodes, S_1 (p nodes) and S_2 (q nodes).

Links can only connect nodes of different classes.



Examples: papers/authors, boards/directors, movies/actors, meetings/persons, reactions/reactants …

A bipartite network is described by the $p \times q$ (rectangular) incidence matrix:

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

An ordinary ("one-mode") weighted network is obtained by projection, e.g., onto the set S_2:

In the projected network, the weight of the link $i \leftrightarrow j$ is the number of neighbors that i, j have in common in S_1 in the bipartite network.

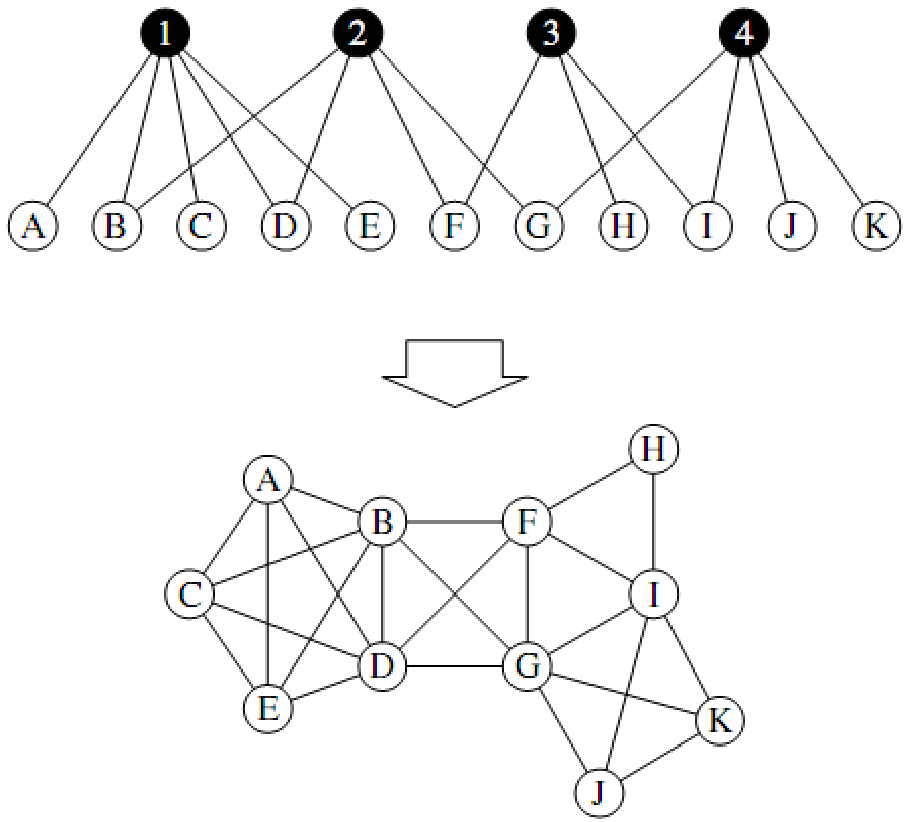
To obtain the weight matrix W of the projected network:

- compute $M = B^T B$
- set the diagonal entries to zero:

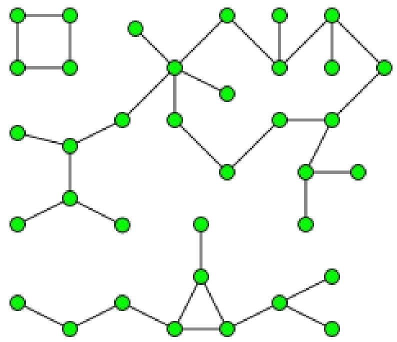
$$W = B^T B - diag(B^T B)$$

In the example:

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 2 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$



COMPONENTS

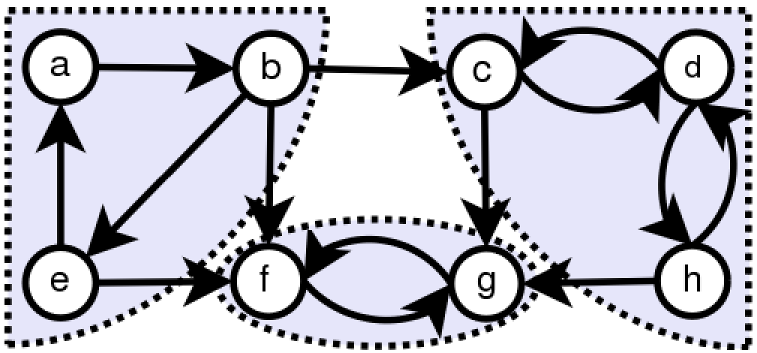


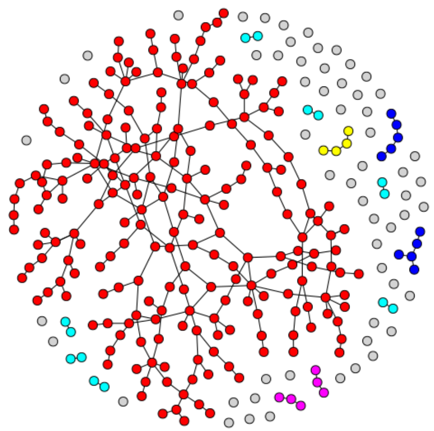
A network is connected if, for any nodes i, j, there is a path $i \rightarrow j$.

A component is a (maximal) connected subnetwork.

A directed network is weakly connected if the undirected network obtained neglecting directions is connected.

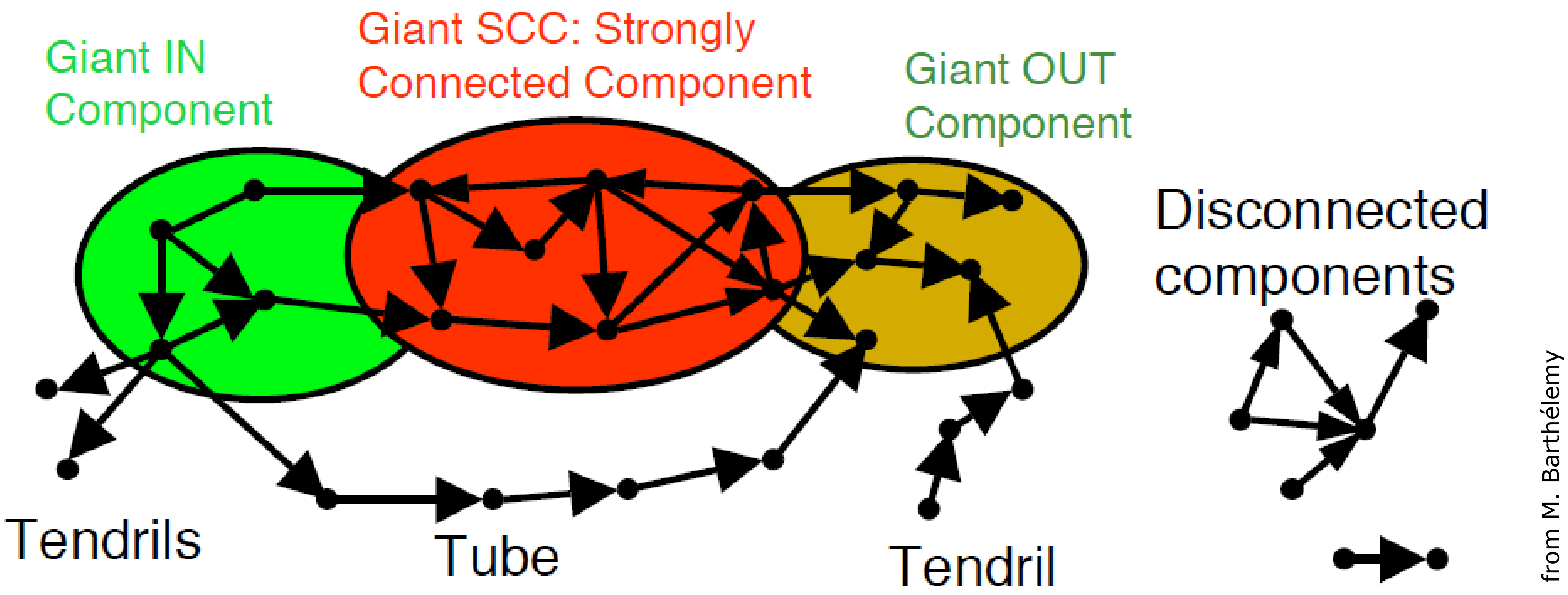
A connected directed network is often denoted as strongly connected, to emphasize the difference with weak connectivity.





A giant component is a component whose size scales with N.

Directed networks call for a more detailed classification. A typical scenario:



SCC: there is a directed path joining any pair of nodes
IN: nodes from which there is a directed path to SCC
OUT: nodes to which there is a directed path from SCC
tendrils and tubes cannot be reached from the SCC

[warning: despite their name the IN and OUT subnetworks are not components]

SCC + IN + OUT + tendrils + tubes = Weakly Connected Component (WCC)